Journal of Petroleum
Science and Technology

A Nonlinear Creep-damage Constitutive Model of Mudstone Based on the Fractional Calculus Theory

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ABSTRACT

During the flood development in an oil field, the creep characteristic of mudstone is one of the important factors causing casing damage. In this study, based on the theory of fractional order differential and taking into account the creep damage evolution rules, a fractional nonlinear creep-damage model is proposed to reflect the instantaneous deformation in loading processes and the accelerated creep stage of mudstone. After assuming that the relationships between strain and time can be expressed by the exponential functions, the creep-damage constitutive model to describe the relationship between stress and strain stress of mudstone is established under the condition of accelerated strain rate loading. Furthermore, uniaxial creep tests and conventional triaxial compression tests were carried out to validate the proposed model. It is found that the fractional nonlinear creep-damage model can effectively describe the characteristics of the three stages of mudstone creep; moreover, this model can reduce the number of components and the parameters. Both the fractional nonlinear creep model and the creep-damage constitutive model have a high fitting relationship with the test results. Also, the initial elastic modulus and confining pressure are in a good linear relationship. Finally, the parametric sensitivity analysis of the theoretical model is carried out. The correctness and applicability of models were confirmed from three respects of the derivation process, test results, and the theoretical analysis.

Keywords: Fractional Calculus, Stress-Strain Relationship, Damage Mechanics, Mudstone Accelerated Creep

INTRODUCTION

Creep behavior is one of the most important mechanical properties of rock. During the flood development in oil field, mudstone creep is an important factor causing casing damage. After the water absorption and softening in mudstone, the mechanical properties and the stress state of which changed, the diagenetic cementation force gradually disappeared, and became plastic. Under the action of far field stress, the mudstone begins to creep and produces displacement and deformation around the borehole that makes the field stress acting on the cement sheath and casing. Therefore, the casing transfigured and damaged [1,2]. An accurate description of the deformation constitutive relation of mudstone is the main problem to solve the casing damage. In recent years, many scholars have made a lot of

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Article history
Received: October 10, 2015
Received in revised form: January 01, 2016
Accepted: February 24, 2016
Available online: November 15, 2017

researches on the rock creep process which mainly consider the first and second creep stage. But in fact, in the process of rock creep, the third stage of creep characteristics has a more important significance [3]. At present, most studies tend to use integer order models to describe the creep behavior and creep constitutive relation of rock and soft soil; these models are intuitive and easy to understand and have clear physical concepts [4-6]. However, a large number of experimental results show that the integer order differential creep model cannot well be consistent with the experimental data. For example, the standard linear creep model can describe the instantaneous and delayed elasticity of solids. But the whole process of the creep compliance and relaxation modulus cannot fit well with the experimental data, especially in the initial stage of creep or relaxation [7,8]. Furthermore, these models cannot describe the whole process curve of rock creep very well, especially for the description of the accelerated creep stage [9-11]. The fractional calculus theory is an extension of the integer order calculus to any order and can be used to study the differential and integral of any order. Therefore, the fractional order calculus theory has obvious advantage in establishing the rock creep constitutive model and describing the rock creep characteristics [12-14]. In recent years, some scholars have used the modeling approach of the element combination model for reference and used the fractional order viscous bodies instead of the classical viscous materials to describe the creep behavior of rock and soil materials. It provides a new idea for the study of the creep model of rock and soil [15,16]. Based on the fractional calculus theory, Yin et al. proposed a kind of software element to simulate the rock and soil material between ideal solid and fluid. The combination model consists of this element and the classical linear mechanical element can describe the nonlinear behavior of the rock decay creep and steady creep [17]. Yin et al. established an improved triaxial creep model of CCG on the basis of a Nishihara model and another visco-elasto-plastic model, parameters of which were fitted on test data [18]. Zhou et al. proposed a new creep constitutive model on the basis of time-based fractional derivative by replacing a Newtonian dashpot in the classical Nishihara model with the fractional derivative Abel dashpot of a variable viscosity coefficient [19,20]. Yao et al. constructed a fractional order derivative Burgers creep model to simulate the steady stage of creep by replacing the serial mode of Burgers model with a fractional order derivative function model; they also used a fractional order derivative acceleration Burger creep model to simulate the acceleration stage of creep by connecting the acceleration element with the fractional order derivative Burgers model [21]. Chen et al. proposed a new four-element nonlinear visco-elasto-plastic rheological model based on an expression with a fractional calculus form [22]. Wu et al. presented a fractional nonlinear dashpot element on the basis of a non-Newton fluid viscous damping element which can reflect the nonlinear acceleration characteristic of the creep curve. Afterwards, a new nonlinear creep constitutive model was obtained by introducing the element [23,24]. Li and Yue built a fractional order derivative nonlinear rheological model which consists of five components and obtained the analytical solution of the model [25]. Kang et al. proposed a fractional non-linear model to describe the creep behavior of coal taking into account the visco-elasto–plastic characteristics and the damage effect [26]. The models in the above research are almost established by replacing the classical element with the fractional order element, which can well fit rock creep test curve of decay creep stage and steady creep stage; however, they cannot reflect the accelerated creep characteristics well.

In this paper, a nonlinear creep-damage model of mudstone with differential expression was proposed combining the damage mechanics theory and the fractional calculus theory. The model has some advantages such as a simple structure and a few numbers of parameters, and it can reflect the whole process of rock creep. After assuming that the relationships between strain and time can be expressed by the exponential functions, the creep-damage constitutive model is established under the condition of accelerated strain rate loading. Finally, the rationality and applicability of the models were verified by the mudstone creep test and the conventional triaxial compression test results.

EXPERIMENTAL PROCEDURES

Establishment of Nonlinear Model

Fractional Calculus Basic Element

Fractional calculus is to extend the order of calculus to the field of fractions and even negative numbers. The software element with fractional calculus is considered to be an ideal element model between solid and fluid, which can well reflect the visco-elastic characteristics of rock and soil materials. Based on the Riemann-Liouville fractional calculus operator theory, the β order integral of the function *f*(t) is defined as:

$$\frac{\mathrm{d}^{-\beta}f\left(t\right)}{\mathrm{d}t^{-\beta}}=D_t^{\beta}f\ (\mathrm{t})=\frac{\mathrm{d}^n}{\mathrm{d}t^n}\left[D_t^{-(\mathrm{n}-\beta)}f\ (\mathrm{t})\right] \tag{1}$$

And the β order differential is defined as:

$$\frac{\mathrm{d}^{\beta}f\left(t\right)}{\mathrm{d}t^{\beta}}=D_t^{\beta}f\ (\mathrm{t})=\frac{\mathrm{d}^n}{\mathrm{d}t^n}\left[D_t^{-(\mathrm{n}-\beta)}f\ (\mathrm{t})\right] \tag{2}$$

where, $\beta>0$, and n-1 < $\beta\le n$;

$\Gamma(\beta)$ is the Gamma function defined as:

$$\Gamma\left(\beta\right)=\int_0^{\infty}e^{-t}t^{\beta-1}dt\ \left(\mathrm{Re}\left(\beta\right)>0\right) \tag{3}$$

The Laplace transformation formula of fractional calculus is:

$$\begin{aligned}L\left[D_t^{-\beta}f\ (\mathrm{t}),\mathrm{p}\right]&=p^{-\beta}\overline{f}\ (p)\\ L\left[D_t^{\beta}f\ (\mathrm{t}),\mathrm{p}\right]&=p^{\beta}\overline{f}\ (p)\end{aligned} \tag{4}$$

(The function *f*(t) can be integrated when *t* is close to 0, $0\le\beta\le1$)

where, $\overline{f}\left(p\right)$ is the Laplace transformation of *f*(t).

According to the classical solid mechanics and the fluid mechanics theory, the constitutive relation of the ideal solid should satisfy Hooke law $\sigma(t)$ - $\varepsilon(t)$, and the ideal fluid should satisfy Newton's law of viscosity $\sigma(t)-d^1\varepsilon(t)/dt^1$. The fractional calculus can be used to study the characteristics of the differential and integral operators of any order. If we transform $\sigma(t)-\varepsilon(t)$ into $\sigma(t)-d^0\varepsilon(t)/dt^0$, the fractional order differential form of the stress-strain relationship of rock mass between the ideal solid and ideal fluid can be expressed as:

$$\sigma\left(t\right)=\xi\frac{\mathrm{d}^{\beta}\varepsilon\left(t\right)}{\mathrm{d}t^{\beta}} \tag{5}$$

where, ξ is the visco-elastic coefficient which is similar to the elastic modulus of Hooke law. The dimension of ξ is "stress-strain β".

When $0<\beta<1$, Equation 5 describes the state of the matter between ideal solid and ideal fluid; When $\beta>1$, Equation 5 describes the state of the accelerating rheology. In this paper, we mainly study the accelerating rheological state of the material. According to this expression method, we define the element which $\beta>1$ as a software element (Figure 1).

Figure 1: Fractional calculus viscous element.

When $\sigma(t)=const$, the stress is a fixed value. Based on the Riemann-Liouville fractional operator theory, Equation 5 can be transformed into:

$$\varepsilon(t)=\frac{\sigma}{\xi}\frac{t^{\beta}}{\Gamma(1+\beta)} \tag{6}$$

Under the condition of constant stress, the creep curves of fractional calculus viscous element are shown in Figure 2. When $\beta=0$ with $\beta=1$, the curves describe the creep characteristics of ideal solid and ideal fluid respectively. When $0<\beta<1$ the strain increases slowly, but the strain rate decreases gradually, showing the characteristic of slow speed change. When $\beta>1$, both strain and strain rate increase significantly with time. Moreover, the creep characteristic is gradually strengthened with an increase in β. By considering the deformation characteristics of three stages of rock creep, it is vivid when $\beta=0$, the fractional order differential element describes the instantaneous elastic deformation. In addition, the element describes the visco-elastic deformation (deceleration creep stage and steady creep stage) when $0<\beta<1$ and visco-plastic deformation (accelerated creep stage) when $\beta>1$.

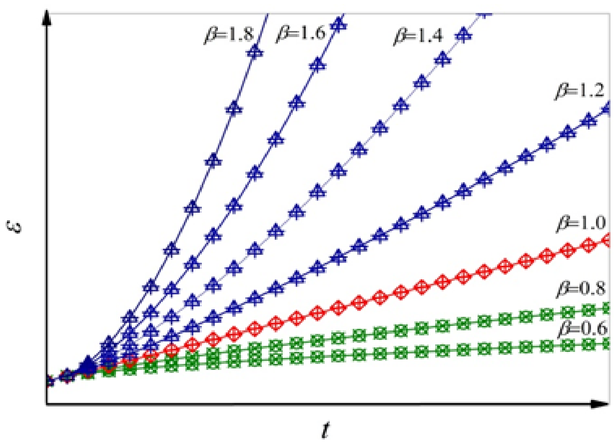


Figure 2: The creep curves of fractional calculus element.

Based on the derivation and the change rule of fractional order model, combining with the typical rock creep deformation characteristics, we can use the element to simulate the creep curve of rock. When *ε(t)=constant* , by using the fractional calculus, the relaxation equation is derived as:

$$\sigma(t)=\xi\,\varepsilon_0\frac{t^{-\beta}}{\Gamma(1-\beta)} \tag{7}$$

where, ε_0 is the initial strain under the initial stress. For the different materials, we can change the creep curve or the relaxation curve by adjusting element parameters β and ξ so as to accurately fit the experimental results of the material.

Damage in the Process of Rock Creep

The critical value of rock creep damage stress is the long-term strength of rock. When the external stress is greater than the long-term strength of rock itself, rock will produce creep damage with an increase in time. According to the basic theory of damage mechanics, under the action of external stress, the micro cracks in the rock mass expand, encrypt, and increase the ability of rock mass to resist damage, and deformation decreases gradually. Therefore, the damage degree of rock mass is related to the magnitude of the external stress and the time. The damage variable in this paper is defined by the method of elastic modulus:

$$D(\sigma,t)=1-\frac{E(\sigma,t)}{E_0} \tag{8}$$

where, E_0 is initial elastic modulus of rock mass; *D(σ, t)* is damage variable of rock mass at time *t*; *E(σ, t)* represents elastic modulus of rock mass at time t and is mainly related to the external stress level at the moment. The rock has lost its carrying capacity after creep and failure. Based on the study on the creep damage of mudstone by Liu et al., *E(σ, t)* is defined using the following relation:

$$E(\sigma,t) = E_0 \exp\left[-\langle \sigma - \sigma_\infty \rangle t/b\right] \tag{9}$$

where, σ_∞ is the long term strength of rock material, and it can be determined by an experiment; *b* is a rock material constant; $\langle \sigma - \sigma_\infty \rangle$ is a step functions defined by:

$$\langle \sigma - \sigma_\infty \rangle = \begin{cases} 0 & (\sigma \le \sigma_\infty) \\ \sigma - \sigma_\infty & (\sigma > \sigma_\infty) \end{cases} \tag{10}$$

After inserting Equation 9 into Equation 8, the following relation is obtained:

$$D(\sigma,t) = 1 - \exp\left[-\langle \sigma - \sigma_\infty \rangle t/b\right] \tag{11}$$

According to Equation 11, when the external load stress is greater than the long-term strength of rock mass $(\sigma > \sigma_\infty)$ and $t \to 0$, the damage variable (*D*) is equal to 1, which means the complete damage of rock. As can be seen, with the increase of time (*t*) and stress (σ), the elastic modulus of the material decreases gradually, but the damage variable increases gradually. Based on the definition of damage variable *D* in Equation 11, the effective stress and nominal stress should meet the following relationships:

$$\tilde{\sigma} = \frac{\sigma}{1 - D(\sigma,t)} \tag{12}$$

where, $\tilde{\sigma}$ is the effective stress, and σ is the nominal stress. Inserting Equation 7 into Equation 8:

$$\tilde{\sigma} = \sigma \exp\left[\langle \sigma - \sigma_\infty \rangle t/b\right] \tag{13}$$

The Fractional Order Creep-damage Model of Mudstone

Due to the phenomenon of elastic deformation of rock mass in the early loading stage, the elastic element is used to describe the instantaneous elastic deformation. The accelerated creep characteristic of rock mass is gradually changing with time and external load stress, and there is a critical value of damage in this process. Therefore, the accelerated deformation process of rock mass is described by using the fractional order damage element. The model structure diagram is shown in Figure 3.

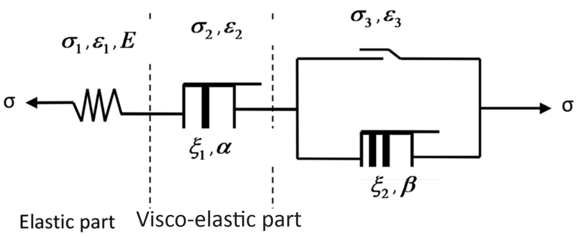


Figure 3: A fractional order damage creep model.

When the external load stress is greater than the long-term strength of rock mass, mudstone is in the stage of accelerated creep. According to the model diagram, we can obtain:

$$\begin{aligned} \sigma &= \sigma_1 = \sigma_2 = \sigma_3 \\ \varepsilon &= \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \end{aligned} \tag{14}$$

$$\begin{cases} \sigma_1 = E\varepsilon_1 \\ \sigma_2 = \xi_1 \dfrac{d^\alpha \varepsilon_2}{dt^\alpha} \\ \sigma_3 = \sigma_\infty + \xi_2 \dfrac{d^\beta \varepsilon_3}{dt^\beta} \end{cases} \tag{15}$$

When the external stress is less than the long-term strength of mudstone, the mudstone is in the stages of creep of I and II, and viscoelastic deformation occurs. In this situation, $0 < \alpha < 1$. After rewriting Equation 15 by Laplace transform and Laplace inverse transform and inserting it into Equation 14, the nonlinear rheological constitutive equation can be obtained:

$$\varepsilon = \frac{\sigma}{E_0} + D_t^{-\alpha}\left(\frac{\sigma}{\xi_1}\right) + D_t^{-\beta}\left(\frac{\sigma - \sigma_\infty}{\xi_2}\right) \tag{16}$$

Under the condition that the external load stress is greater than the long-term strength of the rock mass, the rock mass will enter the stage of creep damage. Equation 16 can be rewritten as:

$$\varepsilon = \frac{\sigma}{E_0} + D_t^{-\alpha}\left(\frac{\sigma}{\xi_1}\right) + D_t^{-\beta}\left(\frac{\tilde{\sigma} - \sigma_\infty}{\xi_2}\right) \tag{17}$$

By considering the initial conditions of *t*=0 and $\sigma = \sigma_0$, the nonlinear creep constitutive equation can be obtained based on the Riemann-Liouville fractional calculus theory.

$$\varepsilon = \frac{\sigma_0}{E_0} + \frac{\sigma_0}{\xi_1}\frac{t^{\alpha}}{\Gamma(1+\alpha)} + \frac{\tilde{\sigma} - \sigma_{\infty}}{\xi_2}\frac{t^{\beta}}{\Gamma(1+\beta)} \tag{18}$$

By inserting Equation 13 into Equation 18, the creep damage constitutive model based on the fractional calculus theory was obtained:

$$\varepsilon = \frac{\sigma_0}{E_0} + \frac{\sigma_0}{\xi_1}\frac{t^{\alpha}}{\Gamma(1+\alpha)} + \frac{\sigma_0 \exp\left[\langle \sigma - \sigma_{\infty} \rangle t/b\right] - \sigma_{\infty}}{\xi_2}\frac{t^{\beta}}{\Gamma(1+\beta)} \tag{19}$$

The Constitutive Model of Mudstone under Accelerated Strain Rate Loading

According to the actual situation, assuming that $\varepsilon = ae^{t}$ and inserting into Equation 19, the stress-strain relationship of mudstone creep under the condition of accelerated strain rate loading can be obtained:

$$\varepsilon = \frac{\sigma}{E_0} + \frac{\sigma}{\xi_1}\frac{(\ln\varepsilon - \ln a)^{\alpha}}{\Gamma(1+\alpha)} + \frac{\sigma \exp\left[\langle \sigma - \sigma_{\infty} \rangle (\ln\varepsilon - \ln a)/b\right] - \sigma_{\infty}}{\xi_2}\frac{(\ln\varepsilon - \ln a)^{\beta}}{\Gamma(1+\beta)} \tag{20}$$

where, *a* is a constant. The creep damage constitutive model of mudstone under the condition of accelerating strain rate loading can be derived by the Equation 20.

$$\sigma = \frac{E_0\xi_1\Gamma(1+\alpha)\xi_2\Gamma(1+\beta)\varepsilon + E_0\xi_1\Gamma(1+\alpha)(\ln\varepsilon - \ln a)^{\beta}\sigma_{\infty}}{\xi_1\Gamma(1+\alpha)\xi_2\Gamma(1+\beta) + E_0\xi_2\Gamma(1+\beta)(\ln\varepsilon - \ln a)^{\beta} + E_0\xi_1\Gamma(1+\alpha)(\ln\varepsilon - \ln a)^{\beta}\exp\left[\langle \sigma - \sigma_{\infty} \rangle (\ln\varepsilon - \ln a)/b\right]} \tag{21}$$

When the external load stress is greater than the long-term strength of mudstone ($\sigma > \sigma_{\infty}$), the mudstone will enter the creep-damage stage and complete damage occurs. At this time, $\exp[-(\sigma - \sigma_{\infty}) t/b] = 0$ and Equation 21 can be rewritten as:

$$\sigma = \frac{E_0\xi_1\Gamma(1+\alpha)\xi_2\Gamma(1+\beta)\varepsilon + E_0\xi_1\Gamma(1+\alpha)(\ln\varepsilon - \ln a)^{\beta}\sigma_{\infty}}{\xi_1\Gamma(1+\alpha)\xi_2\Gamma(1+\beta) + E_0\xi_2\Gamma(1+\beta)(\ln\varepsilon - \ln a)^{\beta} + E_0\xi_1\Gamma(1+\alpha)(\ln\varepsilon - \ln a)^{\beta}} \tag{22}$$

According to Equation 22, the stress-strain curves of mudstone under the creep condition are shown in Figure 4. With an increase in the strain, the stress increases, but the increase rate decreases gradually. It shows that in the process of mudstone creep, the greater the strain is, the greater the stress becomes. The rock stress-strain curves under different fractional orders (β) can also be obtained (Figure 4). As can be seen from the figure, the stress and strain present the exponential relation, and the increasing rate of stress decreases with an increase in β. The smaller the β is, the more obvious the stress changes.

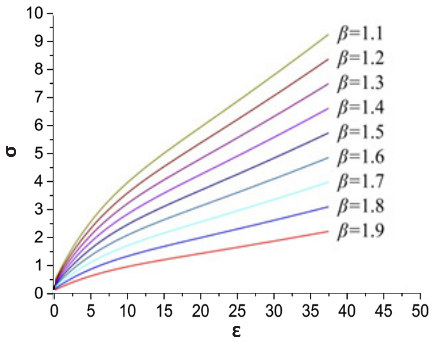


Figure 4: Stress-strain curves from theoretical derivation under different fractional orders (*β*).

Model Validation and a Parametric Sensitivity Analysis

Experimental Verification

In order to verify the correctness and rationality of the models, the creep test and triaxial compression test are carried out. The sample is the mudstone of casing damaged wells from Daqing Oilfield in China. The test is used the experimental set-up of creep test of rock from Northeast Petroleum University (Figure 5). Test instrument parameters are as follows: confining pressure which ranges from 0 to 30 MPa; axial load which ranges from 0 to 600 kN; indoor temperature and humidity which are controlled by air conditioning. The standard size of mudstone samples is 75 mm × 150 mm (diameter × height).

Figure 5: The experimental set-up of the creep test of rock.

First, the creep test of mudstone is carried out by the step loading. The load imposed on the sample increased gradually. The first step loading stress is 4 MPa and then increases by 2 MPa during each step. The loading time of each step is about 14 days and test is stopped until the specimen fails. The whole process creep curves of mudstone were obtained after about 142 days. From the experimental curves in Figure 6, we can see the obvious nonlinear acceleration characteristics at the last loading step, where the loading stress is about 18 MPa. In this step, based on the experimental data, the time to enter the acceleration creep stage can be roughly judged, t=12.34 day and ε_a =5.03%. Based on the nonlinear creep damage model, the experimental data of 8th loading step were fitted and analyzed by using nonlinear least square method. Other initial parameters are assumed as follows: E_0=2.0 GPa; $\xi_1 = 18Gpa.h^{\alpha}, \xi_2 = 18GPa.h^{\beta}, \alpha = 0.3; \beta = 1.5$.

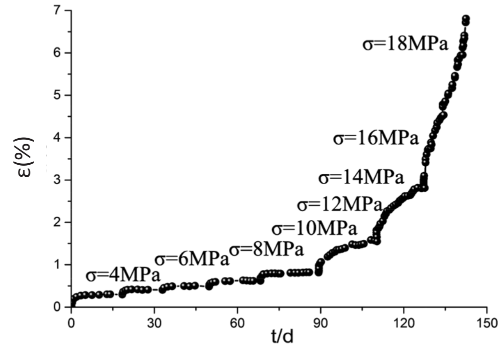


Figure 6: The whole process creep.

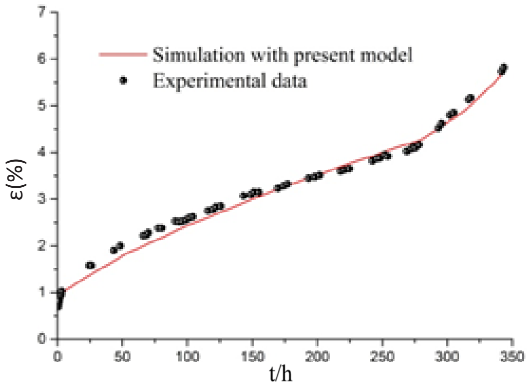


Figure 7: Experimental data and the fitting curves by nonlinear fractional derivative model curves.

The fitting analysis results of the experimental data are shown in Figure 7. It can be seen that the fractional order nonlinear creep-damage model is in good agreement with the experimental data. The fractional order nonlinear creep-damage model can not only reflect the early stages of the creep nonlinear gradual process, but it also describes rock nonlinear accelerative creep properties.

Then, the conventional triaxial compression tests were carried out on the mudstone specimens, and the stress-strain curves at different confining pressures were obtained (Figure 8). Equation 22 is used to fit and analyze the experimental data and found the following features: (1) the experimental results can be well fitted to the constitutive model of mudstone derived in this paper; (2) at different confining pressures, β of each curve is 1.37. It means that β does not change with confining pressure, and it can reflect the "soft and hard" degree of mudstone; (3) at different confining pressures, there is a very good linear relationship between initial elastic modulus and confining pressure of each curve (Figure 9).

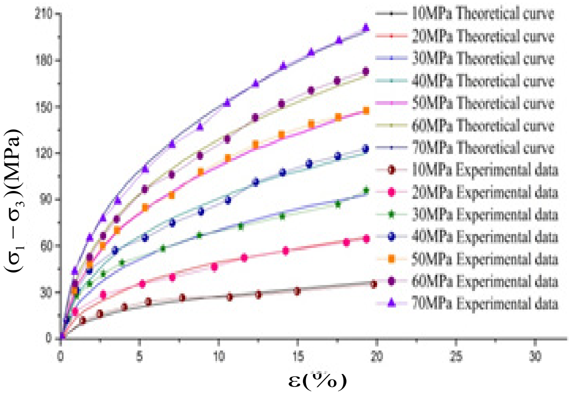


Figure 8: Stress-strain curves of triaxial compression tests.

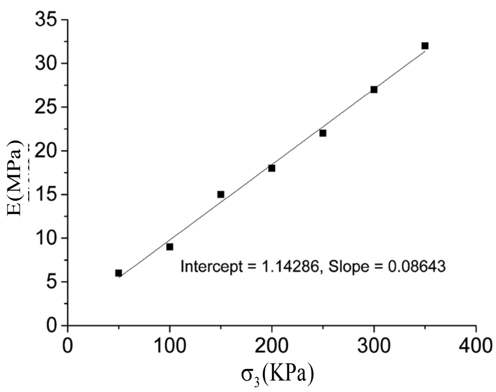
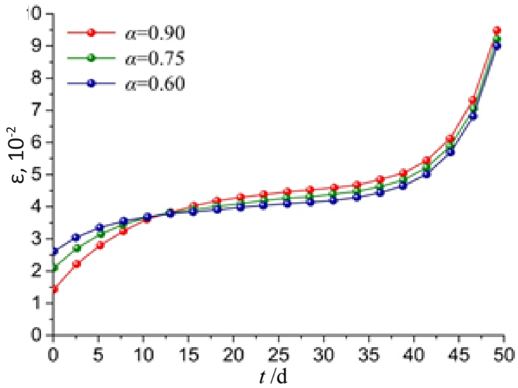


Figure 9: Confining pressure and initial tangential.

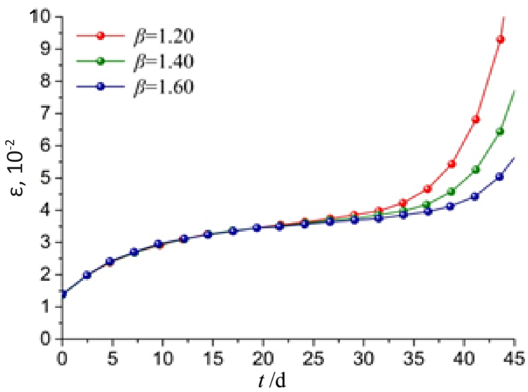
The Parametric Sensitivity Analysis

From the fractional nonlinear creep-damage model of mudstone and the discussion of previous sections, it is obvious that the performance of the proposed model depends on several material parameters, among which the fractional order parameters α and β, the damage factor ω [$\omega=(\sigma-\sigma_\infty)/b$], the fractional viscosity coefficients ξ_1 and ξ_2 are especially important. The sensitivity of creep strain to these parameters is shown in Figures 10-12, in which one parameter takes three different values to show its effect on the strain-time curve of mudstone creep, while other parameters use fixed values.

The effect of fractional order α on the variation of strain with creep time is shown in Figure 10(a). It can be seen that when α increases, indicating the transition of fractional element in the visco-elastic body from an elastic element to a viscous element, the strain rate corresponding to each creep stage rises accordingly. However, the increase of fractional order β only reduces the strain rate corresponding to the accelerating creep stage, while the transient and steady creep stages are scarcely affected, as shown in Figure 10(b). This is because the fractional element in the visco-plastic body takes effect only when the mudstone damage dominates the process of creep deformation.



(a) The effect of α on the mudstone creep curves.



(b) The effect of the β on the mudstone creep curves.

Figure 10: Sensitivity of the creep strain to fractional order parameters.

The effect of damage factor ω [$\omega=(\sigma-\sigma_\infty)/b$] on the creep curves is illustrated in Figure 11. Since the factor ω controls the damage evolution with creep time, the greater the factor ω is, the more rapidly the mudstone damage evolves. Consequently, it can be seen from Figure 11 that the curve corresponding to the larger value of ω branches earlier from the transient creep stage into the accelerating creep stage. It means that the occurrence of accelerating creep is much earlier with an increase in ω. Moreover, the duration of each creep stage is also reduced with an increase in ω.

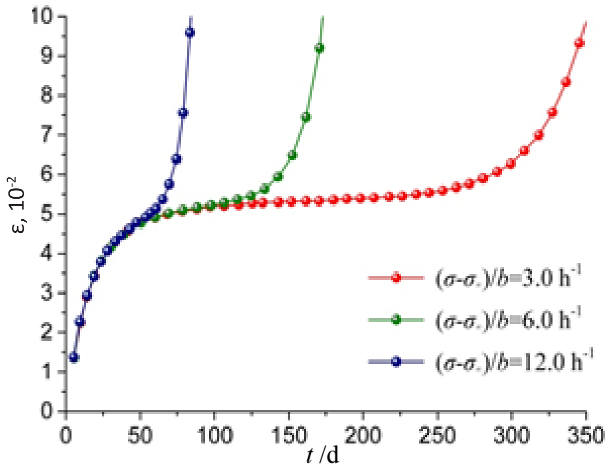


Figure11: The effect of damage factor ω [$\omega=(\sigma-\sigma_\infty)/b$] on the mudstone creep curves.

Finally, the sensitivity of creep strain to fractional viscosity coefficients ξ_1 and ξ_2 is demonstrated in Figure 12. It is apparent that the creep strain increases with the decrease of viscosity. The parameter ξ_1 has a global effect on the creep evolution process, while the effect of ξ_2 is limited to the II and III creep stages, which is similar to the case of fractional orders α and $\boldsymbol{\beta}$. The effect of ξ_1 on the accelerating creep is smaller compared to its effect on transient and steady creep. Furthermore, a smaller value of ξ_2 leads to a shorter period of transient and steady creep.

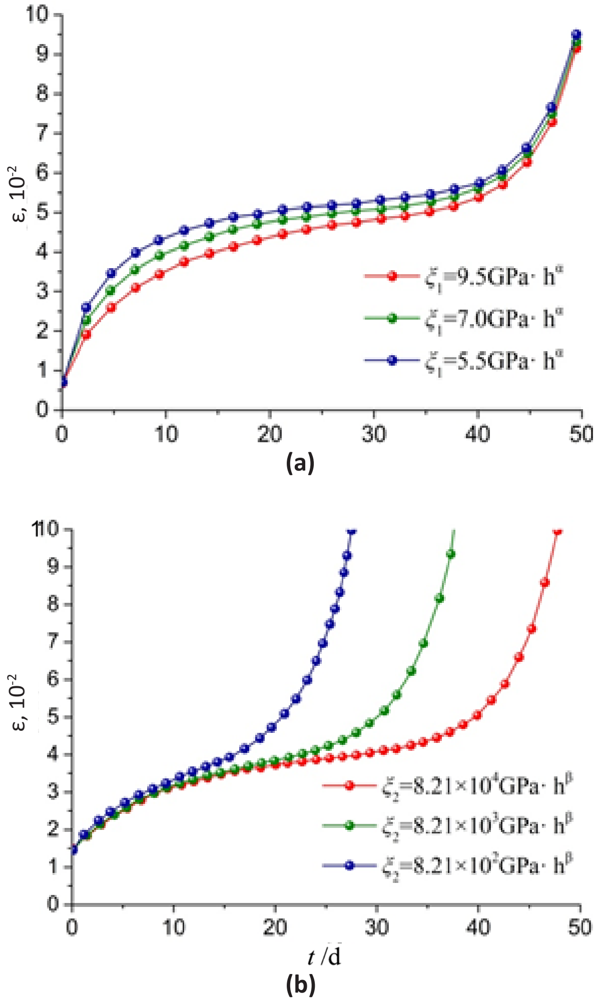


Figure 12: Sensitivity of the creep strain to fractional viscosity coefficients.

CONCLUSIONS

(1) Based on the fractional calculus theory, two software elements with a fractional order differential and a spring element were connected in series. Also, by introducing the creep damage evolution law, a fractional nonlinear creep-damage model of mudstone was proposed which can well describe the instantaneous deformation and the accelerated creep characteristics in loading process.

(2) Based on the nonlinear creep-damage model, assuming that the relationships between strain and time can be expressed by the exponential functions, the creep-damage constitutive model to describe

the relationship between stress and strain stress of mudstone is established under the condition of accelerated strain rate loading. The results show that under the accelerated creep condition, the stress and strain present the exponential relation, and the increasing rate of stress decreases with the increase of β. In addition, the smaller the β is, the more obvious the stress changes.

(3) Through the mudstone creep test, the whole creep curve was obtained. Then, the nonlinear creep-damage model is fitted with the data of creep test. The results show that the model based on fractional differential element can not only reflect the initial creep stage effectively, but it can also reflect the nonlinear acceleration creep characteristics. The model and the experimental data are in good agreement, so this good agreement can reduce the number of components and the parameters.

(4) The creep-damage constitutive model of mudstone is validated by the data obtained from the conventional triaxial compression tests. The results show that the theoretical model is consistent with the experimental data. Moreover, it is found that β does not change with confining pressure, and it can reflect the "soft and hard" degree of mudstone. There is a very good linear relationship between initial elastic modulus and confining pressure.

(5) The fractional calculus theory in describing the mechanical properties of rock and soft soil has other advantages such as the description of the volume change. Due to the limitation of this research, we will next carry out further researches.

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