

Research on Chaos Characteristic of Crack Evolution in Coal-rock Fracturing

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ABSTRACT

Precisely describing the formation and evolution rules of coal-rock fracturing crack have great value on reservoir fracturing improvement and highly efficient mining of coal bed methane well. In this paper, a non-linear dynamic method is used to study crack damage evolution behavior of coal-rock fracturing. Considering distribution characteristics of natural cracks in coal-rock, and based on damage mechanics, a mathematical model on stress around the tip of coal-rock fracturing crack and crack evolution is developed. Micro-crack amounts, circumferential stress, and axial stress at the crack tip of coal-rock fracturing crack evolution process are used as characteristic indicators to describe crack evolutionary. C-C method is chosen to reconstruct the phase space of coal-rock fracturing crack evolution. Correlation dimension, Lyapunov index, and Kolmogorov entropy are introduced as chaos characteristic quantities of crack evolution system, and the process of coal-rock fracturing crack damage evolution could be calculated and described. Coal-rock mass of Zhangchen mining area in Heilongjiang, China was used as a research object, and the results show that as the radial stress increases, the Kolmogorov entropy and the degree of chaos decrease; also, as the circumferential stress increases, the Kolmogorov entropy increases, and a higher degree of chaos is obtained; increasing the number of micro-cracks evolution raises Kolmogorov entropy up to a critical value, and then the Kolmogorov entropy drops, which means the chaotic degree decreases. The results calculated show that crack formation is a damage evolution process which has chaos characteristics. Finally, we provided a new way for further research on coal-rock fracturing crack evolution regularities.

Keywords: Coal-rock Fracturing, Crack Evolutionary, Chaos Characteristic, Nonlinear Dynamics, Zhangchen Mining Area

INTRODUCTION

Coal bed methane (CBM) is a kind of unconventional natural gas which is created inside the coal-rock during historical evolution of geology. It is also a kind of clean and high-quality energy which can be

widely used in industrial fuel, power fuel, chemical fuel, living fuel etc. At present, hydro fracturing is a major technology of CBM production, and more than 90% coal bed methane wells are reconstructed by hydro fracturing.

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Coal-rock layer hydro fracturing is a technology that uses high-pressure pump units to inject high viscosity fracturing fluid into well by a delivery capacity higher than accepting fluid ability of reservoirs, which could generate high pressure. When the pressure is over crustal stress around the wall of the well and reach the tensile strength of the rock, layers will generate cracks. Because of the development of the coal-rock crack system, weak structural planes (like bedding plane, cleat face, secondary joint surface etc.) at all levels of coal-rock hydro fracturing crack start breaking, extending and widening, which leads to a very complicated network crack system [1-3]. Crack sizes are different in the crack network system, and crack distributions are confusing and complicated, so description is limited.

Researchers at home and abroad have carried out some research on coal-rock fracturing. Olovyanyn et al. [4] set up the mathematical model of coal-rock hydro fracturing, and the initiation fracturing pressure, and the direction of coal-rock hydro fracturing are obtained by calculating; Lekontsev et al. [5] used Bereznovskaya coal mine as a research object and performed research on the initiation fracturing and expanding regulations of oriented fracturing crack; Li Tonglin et al. [6] researched into basic theories like basic mechanic properties of coal-rock, crack forming conditions of coal-rock hydro fracturing, crack patterns, cracking angle positions etc. Huang Bingxiang et al. [7] set up the calculation model on cleating begin to fracturing of coal-rock hydro fracturing base on fracture mechanics and found the minimum initiation pressure of coal-rock hydro fracturing and crack extension length; Li Chengcheng and Pan Yishan [8] combine theories with numerical simulation to talk about the influences of different crustal stresses on the pressure of cleating begin to fracturing of coal-rock hydrofracturing and

crack expanding state, set up a mechanical model, and analyze the direction and shape of cracks of coal-rock at different crustal stresses; the results show that the initiation pressure is connected with crustal stress difference, and the more the crustal stress difference becomes, the less the initiation pressure is. Ding Jinli et al. of Tsinghua University [9] studied the necessary conditions of sample hydro fracturing destructions by thin and thick walled cylindrical specimen and soil materials; Chen Mian et al. [10] simulated formation conditions by system of large-sized true triaxial simulation test, and used cement mortars as the experimental materials to study the trends and changes of crack width of hydro fracturing cracks by simulation. Deng Guangzhe et al. [11] used coal samples of Tongchuan mine area to study the controlling parameters of crack expanding behaviors, permeability changes of coal samples, and crack expanding behaviors under complicated stresses.

The development of chaos in recent years provides a theoretical basis for research on rock nonlinear characteristics. Peng Tao and He Manchao [12] studied the chaos characteristics in the process of coal soft rock deformation. Yin Guangzhi et al. [13-15] conducted a series of researches on rock microscopic fracture process and bifurcation and chaos characteristics in micro-crack evolution by nonlinear theories, and they discussed the nonlinear evolution characteristics of rock burst occurrence mechanism. Zhou Hui et al. [16] studied chaos characteristics in the process of mining quake by physical cellular automate model, and set up the nonlinear prediction techniques of mining quake system. Song Weiyuan et al. [17] proposed the methods on the prediction and forecasting of rock burst in a certain amount of time scale, which was based on studying the chaos characteristics of rock

burst. Liu Chuanxiao et al. [18] used chaos dynamics to analyze the evolutionary process of the complete stress-strain system from a high chaotic state to a low chaotic state. Li Xibing et al. [19-20] studied the regularities of rock acoustic emission and the situations of chaotic attractors based on a chaos theory, and put forward corresponding prediction methods. Zhu Fangcai et al. [21] used rock-testing machine as a rock mechanics system, and researched chaotic behaviors in rock uniaxial compression process. Jiang Yongdong et al. [22] studied the chaos characteristics in a complete stress-strain system based on the evolutionary process of acoustic emission hits. Ai Chi et al. [23] discovered the chaos characteristics in the research of rock hydrofracturing process, and characteristics of micro-crack evolution. Tao Hui et al. [24] studied the chaos characteristics in the process of rock burst destruction based on micro-earthquake time series. This paper studies the distribution characteristics of natural fracture in coal-rock mass and sets up the basic model of coal-rock fracturing crack evolution by choosing micro-crack amounts, circumferential stress and axial stress at the crack tip of coal-rock fracturing crack evolution process as characteristic indicators to describe crack evolutionary. C-C method is chosen to reconstruct the phase space of coal-rock fracturing crack evolution. Correlation dimension and introducing Lyapunov index and Kolmogorov entropy as chaos characteristic quantities of crack evolution system, and then the process of coal-rock fracturing crack damage evolution could be calculated and described. Coal-rock mass of Zhangchen mining area in Heilongjiang, China is used as a research object, the chaotic characteristic parameters of the three characteristic data are calculated to analyze the chaotic characteristic of crack evolution. The calculated results show that coal-rock fracturing

crack evolution has a chaos characteristic, and crack formation is a damage evolution process, which has chaos characteristics. The current work provides a new way for further research on coal-rock fracturing crack evolution regularities.

THE MODEL OF COAL-ROCK FRACTURING CRACK EVOLUTION

The Mathematical Models of Crack Tip Stress

Describing fracturing micro-crack evolution as a dynamic process in which micro-cracks nucleate, expand, and connect, it deals with crack patterns and expansion rates. The radial damage stress of nonlinear tensor near the crack tip is given by:

$$\begin{aligned} & n_k^{(s)} n_l^{(s)} e_k^{(s)} e_l^{(s)} : \sigma_{ij}^{rdyn(s)} \\ &= \frac{K_1^{dyn(s)}(i) \tilde{l}}{\sqrt{2\pi}(\tilde{l} + a)} \sqrt{\frac{1}{r_1}} \\ & - n_k^{(s)} n_l^{(s)} e_k^{(s)} e_l^{(s)} : \sigma_{ij}^{h(s)} \end{aligned} \quad (1)$$

$$\begin{aligned} & n_k^{(s)} n_l^{(s)} e_k^{(s)} e_l^{(s)} : \sigma_{ij}^{\theta dyn(s)} \\ &= \frac{1 + \sin \phi}{1 - \sin \phi} \left(n_k^{(s)} n_l^{(s)} e_k^{(s)} e_l^{(s)} : \sigma_{ij}^{rdyn(s)} \right) \\ & + n_k^{(s)} n_l^{(s)} e_k^{(s)} e_l^{(s)} : \sigma_{ij}^c \end{aligned} \quad (2)$$

Here $n_k^{(s)}$ and $n_l^{(s)}$ are the groups divalent tensors of crack plane position; $e_k^{(s)}$ and $e_l^{(s)}$ are unit basic vector on the axis direction; $\sigma_{ij}^{rdyn(s)}$ is radial apparent stress at the crack tip of coal-rock fracturing; $K_1^{dyn(s)}(i)$ is dynamic stress intensity factor, and r_1 is the distance from crack tip; m is the apparent stress on the vertical direction of the i th crack surface; $\tilde{l}$ is the average characteristic value in forming micro-crack, and a is the length of increasing micro-crack; σ_{ij}^c is the expression forms of residual stress during rock damage.

Hypothesis crack evolution deals with rock mass characteristics around cracks and evolution rate by defining dynamic stress intensity factor $K_I^{\text{dyn}(s)}(i)$ of crack evolution as reads:

$$K_I^{\text{dyn}(s)}(i) = [k_1 K_d^{(s)}(i) + k_2 K_v^{(s)}(i)] K_I \quad (3)$$

where

$$\left\{ \begin{aligned} K_d^{(s)}(i) &= \frac{1}{\sqrt{I - \ln \frac{A+B}{I - \sum_{s=1}^{N_2} \int_v n_{0f}^{(s)} \otimes n_{0f}^{(s)} dv_0^{f(s)}}}} \\ K_v^{(s)}(i) &= \frac{v_e - v}{\sqrt{v_e^2 - 0.919 \frac{\sqrt{2(1-2\mu)} v_e}{\sqrt{1-\mu}} \left(1 - \frac{\sqrt{1-2\mu}}{\sqrt{2(1-\mu)}} \right)^2 v}} \\ K_I &= (p_{ij}^{(s)}(i) - n_k^{(s)} n_l^{(s)} e_k^{(s)} e_l^{(s)} : \sigma_{ij}^{h(s)}) \sqrt{\pi a} \end{aligned} \right. \quad (4)$$

$$n_i = F_1(\sigma) \cdot (n_0 + m_0) + F_2(\sigma) \cdot (N - (n_0 + m_0)) \cdot \rho_i \quad (5)$$

where, N is the total number of micro cracks in effective growing zone, and ρ_i is the density of micro cracks defined as $\rho_i = (n+m)/N$; F_1 is the survival rate of micro cracks (generally 0.8-0.84), and $F_2(\sigma)$ is the impact capability of fracturing loading stress, zero dimension. As for sandstone fracturing development, we choose:

$$F_2(\sigma) = \frac{1}{(1-D)} \frac{\ln \sigma}{\ln \sigma_c} \quad (6)$$

Substituting Equation 9 into Equation 8 gives:

$$\begin{aligned} n_i &= F_1 \cdot (n_0 + m_0) + \left[\frac{1}{(1-D)} \frac{\ln \sigma}{\ln \sigma_c} \right] \\ (N - n_0 - m_0) \frac{n_0 + m_0}{N} \end{aligned} \quad (7)$$

where, σ is hydro fracturing loading stress. Thus, one may obtain:

$$\begin{aligned} n_i &= F_1(n_0 + m_0) + \frac{1}{(1-D)} \frac{\ln(\delta p_f(t) - \sigma_\theta)}{\ln \sigma_c} \\ (N - n_0 - m_0) \frac{n_0 + m_0}{N} \end{aligned} \quad (8)$$

where, D is the damaging variable.

Phase Space Reconstruction of Coal-Rock Fracturing Crack Evolution

Crack evolution is a complicated nonlinear dynamic system; its mathematical model is difficult to drive by limited variables. To analyze whether the crack system has chaos characteristics, we need to figure out the following characteristic quantities: correlation dimension, Lyapunov index, Kolmogorov entropy etc. These characteristic quantities need to be calculated in phase space, so we need to reconstruct phase space at first. In this essay, we choose time delay reconstruction method to study the chaos characteristics of cracks, and we use time series $\{x(t)\}$ to reconstruct the phase space of crack evolution dynamic system.

Hypothesis n -dimension ($n \geq m$) phase space of crack evolution dynamic system is expressed as:

$$x^{(n)} = f_i(x, x', \dots, x^{(n-1)}) \quad (9)$$

where, m is embedded dimension, and x', s ($i=1, 2, \dots, n$) are variables that participate in crack evolution dynamic system.

Time-evolution form described in Equation 12 is given by:

$$X(t) = (x(t), x'(t), \dots, x^{(n-1)}(t)) \quad (10)$$

We can use discrete and difference equations to replace all-order derivatives as given in Equation 13:

$$X(t) = (x(t), x(t+\tau), \dots, x(t+(n-1)\tau)) \quad (11)$$

Equation 14 is the reconstruct phase space based on time delay reconstruction method, and τ is delay time. M and τ are major parameters in phase space reconstruction system of crack evolution. If τ is too small, two independent coordinate components could not be offered, thereby leading to information redundancy. If τ is too large, two coordinates are completely independent in a statistical sense, and the track of chaos attractor in two directions projection showed no correlation, named "uncorrelation". If embedded dimension m is too small, attractors could be folded and even self-intersected, which leads to reconstructed attractors totally different from the original attractors. If m is too large, noise will be amplified and the calculated amount will be increased, which is undesirable. Therefore, in order to guarantee the quality of reconstructed phase space, we need to use a proper method to get the value of m and τ . C-C method is used to reconstruct phase space. Using the hypothesis of time series $(\delta_1, \delta_2, \dots, \delta_i, \dots, \delta_N)$ of coal-rock fracturing system, phase space can be reconstructed by C-C method:

$$X_i = [\delta_i, \delta_{i+\tau}, \dots, \delta_{i+2\tau}, \dots, \delta_{i+(m-1)\tau}]^T$$

$$(i = 1, 2, \dots, N - (m-1)\tau) X_i \in R^m \quad (12)$$

where, t ($t=1, 2, \dots, n$) is the delay time index.

Correlation integral of time series is defined by:

$$C(m, N, r, t) = \frac{2}{M(M-1)} \sum_{j=1}^M \sum_{i \neq j+1}^M \Theta(r - \|X_i - X_j\|) \quad (13)$$

where, $r > 0$ and $\Theta(a) = \begin{cases} 0 & a < 0 \\ 1 & a > 0 \end{cases}$; $C(m, N, r, t)$ is Probability of distance between two points less than r in phase space; N is the size of data set, and $M = N - (m-1)t$ is phase amount in m -dimension phase space; $\|\dots\|$ is Euclidean distance. The limit of $C(m, N, r, t)$ is $C(m, r, t)$.

Introducing function $S_1(m, N, r, t)$ defines monitoring statistics as:

$$S_1(m, N, r, t) = C(m, N, r, t) - C^m(1, N, r, t) \quad (14)$$

Determining the delay time is to investigate the dependence of $S_1(m, N, r, t)$ to delay time index. In the actual calculation, time series $\{\delta_i\}$ ($i=1, 2, \dots, N$) is divided into t equal non-intersect subsequences, i.e.:

$$S_2(m, r, t) = \frac{1}{t} \sum_{s=1}^t [C_s(m, r, t) - C_s^m(1, r, t)] (m = 2, 3, \dots) \quad (15)$$

Numerical experiments indicate that, when $2 \leq m \leq 5$, $\theta/2 \leq r \leq 2\theta$, and $N \geq 500$, $S_2(m, N, r, t)$ can represent the correlation of the series, where σ is the mean squared error of the dataset. By setting $m=2, 3, 4, 5$; $r_i=0.5\sigma$; and $i=1, 2, 3, 4$, one may define the following average quantities:

$$\bar{S}_2(t) = \frac{1}{16} \sum_{m=2}^5 \sum_{i=1}^4 S_2(m, r, t) \quad (16)$$

$$\Delta \bar{S}_2(k) = \frac{1}{4} \sum_{m=2}^5 \Delta S_2(m, t) \quad (17)$$

$$S_{2cor}(k) = \Delta \bar{S}_2(t) + \left| \bar{S}_2(t) \right| \quad (18)$$

The first zero point of $\bar{S}_2(k)$ or the first minimal value of $\Delta \bar{S}_2(k)$ are selected as the optimum time delay. Together with $\bar{S}_2(k)$ and $\Delta \bar{S}_2(k)$, one may find the global minimal value of $S_{2cor}(k)$ as the length of time window of time series.

Certainty Test of Time Series

The complexity of time series of coal-rock fracturing crack evolution has external random factors and its own internal factors of deterministic dynamics. Therefore, it is necessary to conduct certainty tests of time series of crack evolution.

By setting Y_0 as a fixed vector in R_m , $Y_1, Y_2, \dots, Y_i$ represent i neighboring points in phase space, and $Z_1, Z_2, \dots, Z_i$ are corresponding mapping values. Transfer vector is defined as:

$$V_i = Y_i - Z_i \quad (i = 0, 1, \dots, l) \quad (19)$$

Transfer error is given by:

$$E_{error} = \frac{1}{l+1} \sum_{i=0}^l \frac{\|Y_i - \langle V \rangle\|^2}{\|\langle V \rangle\|^2} \quad (20)$$

where, $\langle V \rangle$ is the mathematical expectation of the transfer vector.

Calculation of Correlation Dimension

Reconstructing phase space R^m by time series, one can calculate correlation integral:

$$C(m, r) = \frac{2}{M(M-1)} \sum_{j=1}^M \sum_{i \neq j+1}^M \Theta(r - \|X_i - X_j\|) \quad (21)$$

Variables in Equation 24 have the same meaning as the one in Equation 16.

As for the fractal structure, because of its self-similarity characteristic, an interval of r has the following scale relation:

$$C(m, r) \propto r^\gamma \quad (22)$$

We can affirm that the index γ in Equation 25 is a dimension, too. Actually, γ is approaching correlation dimension. Correlation dimension is defined by:

$$D_2 = \lim_{r \rightarrow 0} \frac{\ln C(m, r)}{\ln r} \quad (23)$$

where, D_2 is correlation dimension.

Calculation of Lyapunov Index

Lyapunov index, is an essentially quantitative index to measure the process of crack evolution of coal-rock fracturing. If the maximum Lyapunov index is positive, the system is chaotic, otherwise the system remains steady. As for the calculation of Lyapunov index, this paper adopts improved small-data method based on Wolf [25]. The main calculating processes are:

(1) Reconstructing phase space R^m by time series of crack system:

$$X_i = [\delta_i, \delta_{i+t}, \delta_{i+2t}, \dots, \delta_{i+(m-1)t}]^T \quad (i = 1, 2, \dots, N - (m-1)t) \quad X_i \in R^m \quad (24)$$

(2) Searching every X'_j nearest to X_j in phase space and limit them to short separation, that is:

$$d_j(0) = \min \|X_j - X'_j\|, |j - j'| > P \quad (25)$$

where, P is the average period of time series.

(3) For every X'_j in phase space, calculating distance $d_j(t)$ after t discrete time steps:

$$d_j(t) = \min \|X_{i+1} - X'_{i+1}\| \quad t = 1, 2, \dots, \min(M-j, M-j') \quad (26)$$

(4) For every t , calculating average $x(t)$ of $\ln d_j(t)$ of all j :

$$x(t) = \frac{1}{q\Delta t} \sum_{j=1}^q \ln d_j(t) \quad (27)$$

where, q is the amount of nonzero $d_j(t)$.

(5) Drawing the curve of $x(t)$ - t , from the straight slope of which the maximum Lyapunov index λ can be obtained via least square method.

Calculation of Kolmogorov Entropy

Kolmogorov entropy can evaluate chaos extent of the chaos system quantitatively [26]. $K=0$ means system is regular, and $K>0$ means the crack evolution has chaos characteristics. The more complex the crack evolution system is, the larger the K entropy is, and the higher chaos extent becomes.

Considering strange attractor orbits $\{x_1(t), x_2(t), \dots, x_m(t)\}$ of an m -dimension dynamic system, set phase space is divided into im size boxes, and the system condition can be observed in τ time interval. Set $P(i_1, i_2, \dots, i_m)$ is the joint probability of $X(t=\tau)$ in i_1 box ... $X(t=m\tau)$ in i_m box. Thus, Kolmogorov entropy is given by:

$$K = -\lim_{\Delta t \rightarrow 0} \lim_{\varepsilon \rightarrow 0} \lim_{m \rightarrow \infty} \frac{1}{m\tau} \ln \sum_{i_1, \dots, i_m} P(i_1, i_2, \dots, i_m) \log_2 P(i_1, i_2, \dots, i_m) \quad (28)$$

For the crack evolution chaos system of coal-rock fracturing K_2 is not zero ($K_2 \neq 0$). Therefore, we can estimate K_1 in virtue of 2-steps Renyi entropy. 2-steps Renyi entropy is defined as:

$$K_2 = -\lim_{\Delta t \rightarrow 0} \lim_{\varepsilon \rightarrow 0} \lim_{m \rightarrow \infty} \frac{1}{m\tau} \ln \sum_{i_1, \dots, i_m} P^2(i_1, \dots, i_m) \quad (29)$$

Correlation integral $C(m, r)$ has the following relationship:

$$C(m, r) \approx \sum_{i_1, \dots, i_m} p^2(i_1, \dots, i_m) \quad (30)$$

Through setting and analyzing, one may obtain:

$$K_2 = -\lim_{\Delta t \rightarrow 0} \lim_{\varepsilon \rightarrow 0} \lim_{m \rightarrow \infty} \frac{1}{m\tau} \ln C(m, r) \quad (31)$$

By select a proper delay time τ to reconstruct m -dimension phase space, we obtain:

$$K_2(m, r) = \frac{1}{\tau} \ln \frac{C(m, r)}{C(m+1, r)} \quad (32)$$

set phase space is divided into im size boxes, and the system condition can be observed in τ time interval. Set $P(i_1, i_2, \dots, i_m)$ is the joint probability of $X(t=\tau)$ in i_1 box ... $X(t=m\tau)$ in im box. Thus, Kolmogorov entropy is given by:

$$K_2 = \lim_{\substack{m \rightarrow \infty \\ \tau \rightarrow 0}} K_2(m, r) \quad (33)$$

Example Verification of Coal-Rock Crack Evolution

Coal sample for petrology is taken from right six alley, 3# layer, west three area, Zhangchen mining area, Jixi, Heilongjiang. Buried depth is 901-910 m. The sample size is 400×300×180 mm, and the development density of butt cleat ranges 4-5 pieces/10 cm (Figure 1); the density of face cleat ranges 11-12 pieces/10 cm (Figure 2), and the slope angle of cleat is 16°; the length of natural crack ranges 3-6 cm, and crack width is 1.26 mm. The sample size of coal-rock mechanics parameter test is $\Phi 25 \times 50 \sim 75$ mm and $\Phi 50 \times 25 \sim 50$ mm (Figure 3); the surrounding rock pressure is 10 mPa. The sample size of coal-rock mechanics is 300×300×300 mm (Figure 4). The relationship between the net pressure in crack and acoustic emission is shown in Figure 5.

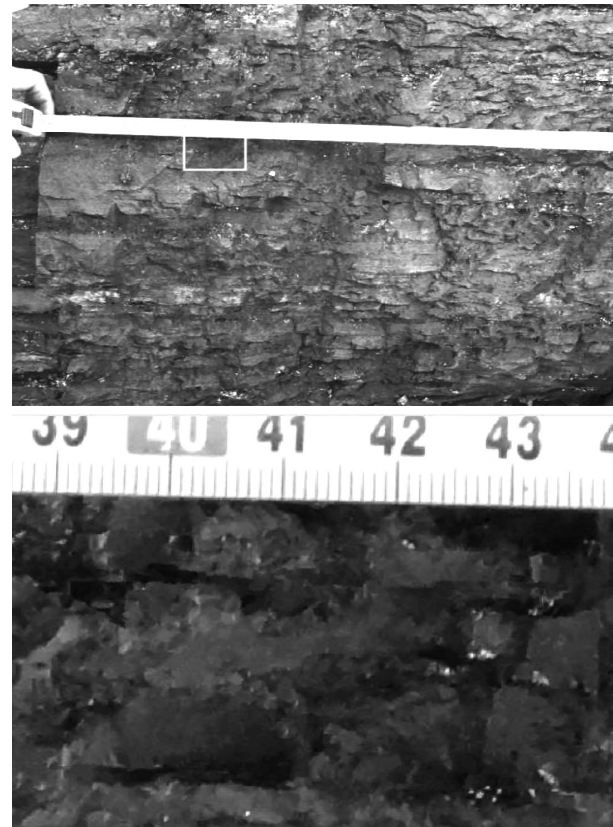


Figure 1: Cleat feature of coal vertical to face cleat direction in Right 6 Alley.

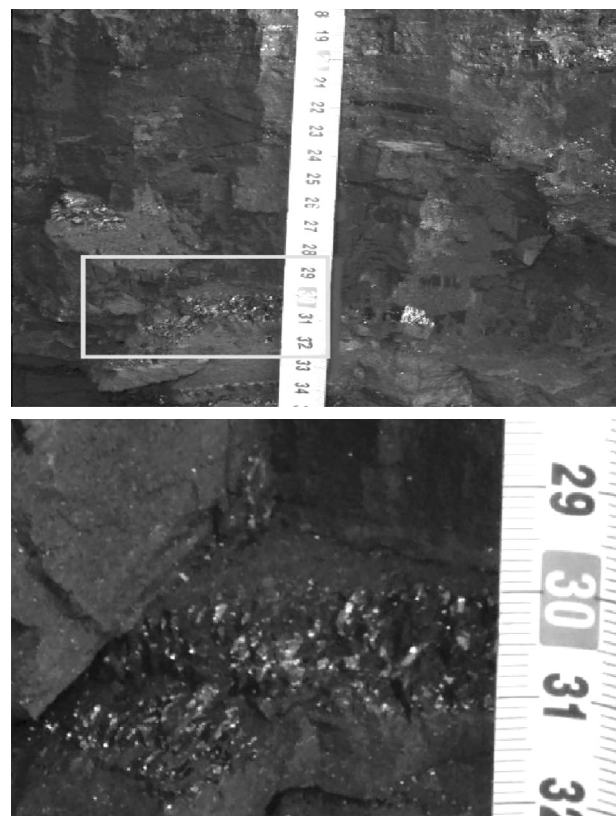


Figure 2: Cleat feature of coal vertical to butt cleat direction in Right 6 Alley.



Figure 3: The coal sample of compression test (Φ25).



Figure 4: Experiment core for fracturing (300mm×300mm×300mm).

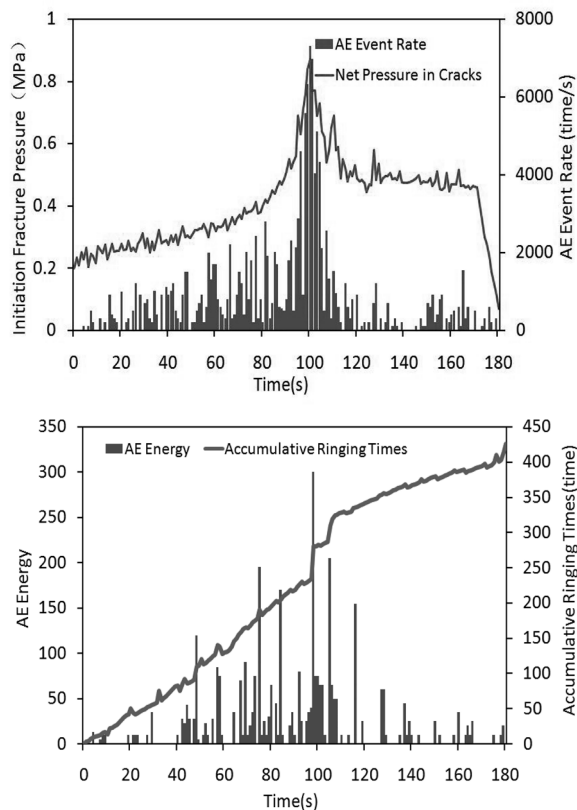


Figure 5: The relationship between the net pressure in crack and acoustic emission.

Reconstruction Parameters

Figures 6-8 display curves of $\bar{S}_2(t)$, $\Delta\bar{S}_2(t)$ and $S_{2cor}(t)$ as a function of delay time index t ; it is clear that that the first time $\Delta\bar{S}_2(t)$ reaching the minimum value is earlier than the time $\bar{S}_2(t)$ reaching zero, and the delay time is equal to 3 ($t=3$). When $t=13$, $S_{2cor}(t)$ reaches the minimum value, a delay time window of $13\tau_s$ ($\tau_w=13\tau_s$), and the minimum embedding dimension $m=5$.

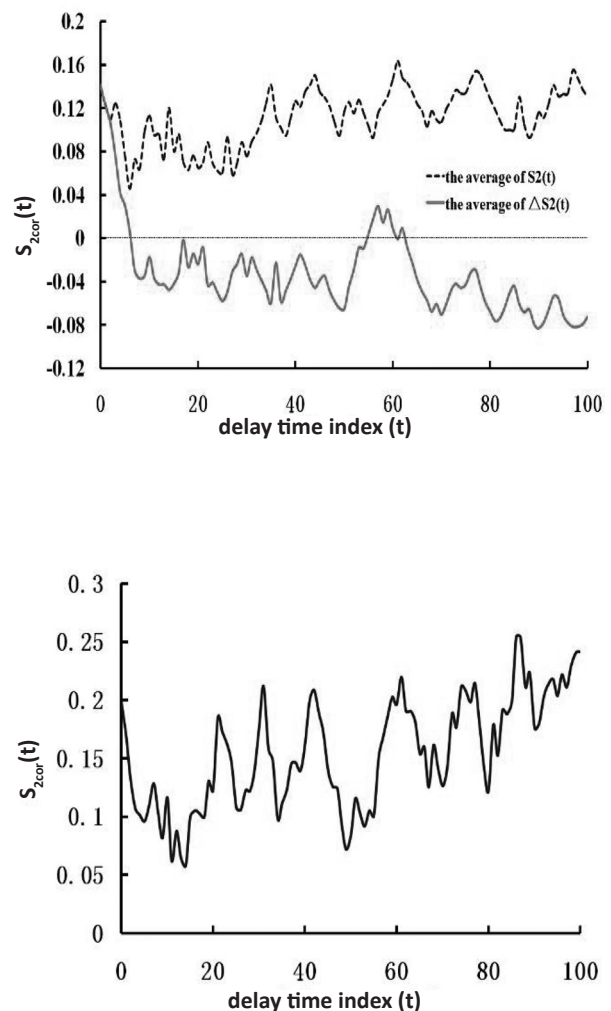


Figure 6: The variation of the number of micro cracks $S_2(t)$, $\Delta\bar{S}_2(t)$ and $S_{2cor}(t)$ with delay time index (t).

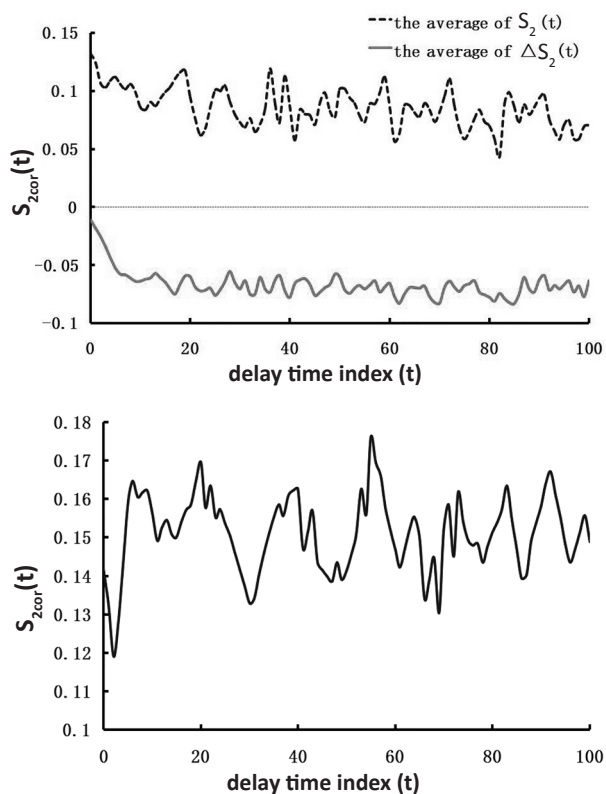


Figure 7: The variation of the circumferential stress $\bar{S}_2(t)$, $\Delta\bar{S}_2(t)$, and $S_{2cor}(t)$ with delay time index (t).

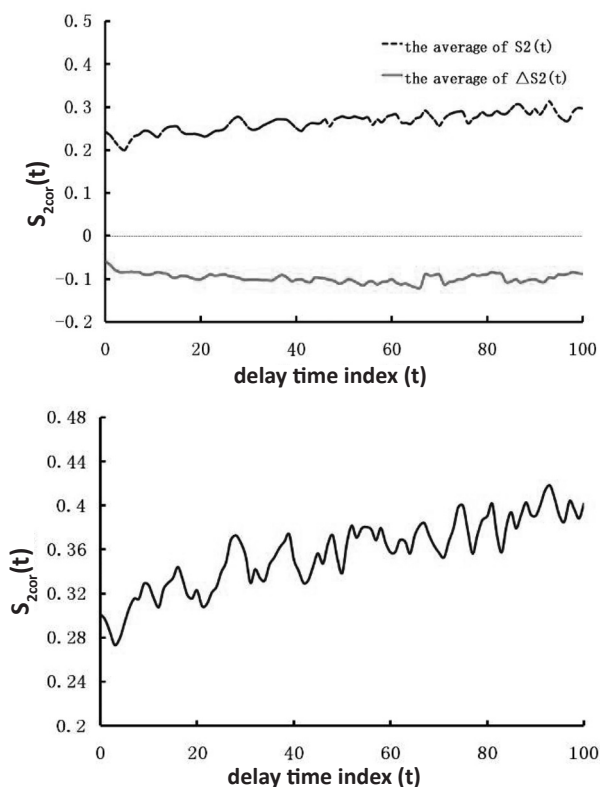


Figure 8: The variation of the radial stress $\bar{S}_2(t)$, $\Delta\bar{S}_2(t)$ and $S_{2cor}(t)$ with delay time index (t).

We then need to reconstruct micro-crack amount series, the phase space of circumferential stress, and the axial stress time series at the crack tip, and choose embedded dimension $m=1,2,\dots,12$. $i(m,r)$ can be calculated by plotting curve $\ln C(m,r)$ versus $\ln r$ as follows:

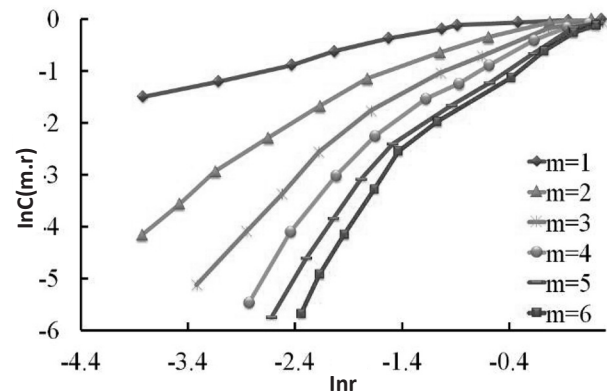


Figure 9: Micro crack amount time series curve $\ln C(m,r)$ - $\ln r$.

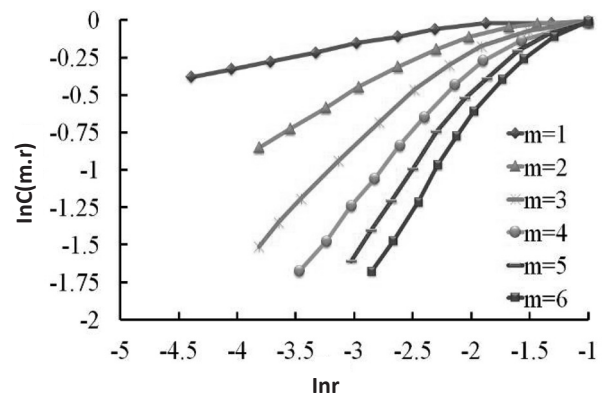


Figure 10: Crack tip circumferential stress time series curve $\ln C(m,r)$ - $\ln r$.

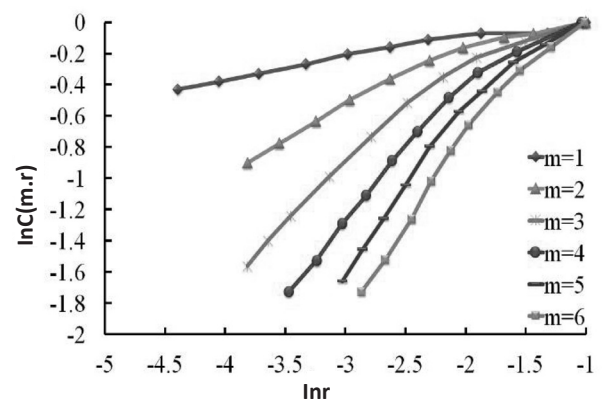


Figure 11: Crack tip axial stress time series curve $\ln C(m,r)$ - $\ln r$.

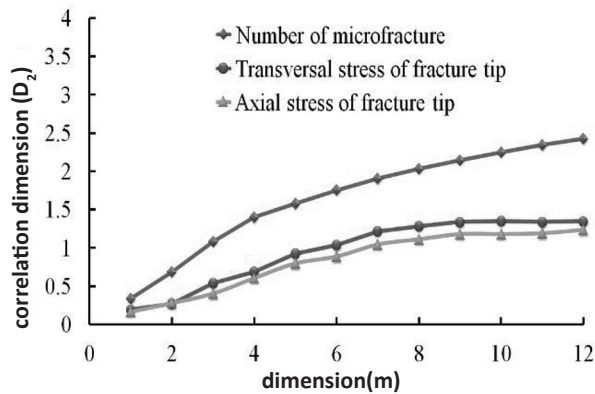


Figure 12: Correlation dimension (D_2) variation with embedded dimension m .

From Figures 9-11, we can see obvious straight line segments in curve $\ln C(m, r)$ versus $\ln r$. When m is small, the slopes of straight line segments are small and the distances between lines are large. The slopes increase and distances decrease and gradually move closer with the increase of m . When $m \geq 5$, the straight line segments intend to be in parallel with each other and become increasingly intensive. The slopes do not change and can be considered as a fixed value, which is defined as the correlation dimension as a characterization of these series.

Figure 12 shows the variation of correlations dimension (D_2) versus embedded dimension (m). The correlation dimensions of these three series are 2.12, 1.34, and 1.23. Due to their non-integer, we can preliminary concluded that the time series of coal-rock fracturing crack system contains chaotic components.

Calculation of Lyapunov Index

Figures 13-15 show the variation of $x(t)$ versus time (t) evolution for various when embedding dimensions $m=3, 4, 5, 6$, and 7 . From Figures 13-15 we can infer that curves are all increasing functions and the maximum Lyapunov index is positive, which means adjacent tracks are divergent; Moreover, they illustrate that the system has chaotic characteristics. For those curves close to straight lines after embedding dimension, the

least square fitting method is used to handle the part. We obtained the maximum Lyapunov index of three time series respectively equal to 0.0625, 0.0063, and 0.023.

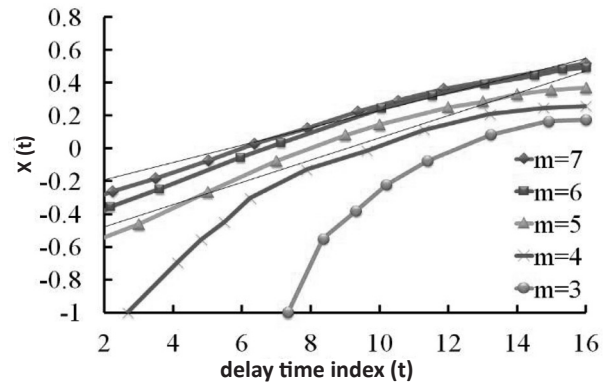


Figure 13: Maximum Lyapunov index of micro crack as a function of delay time.

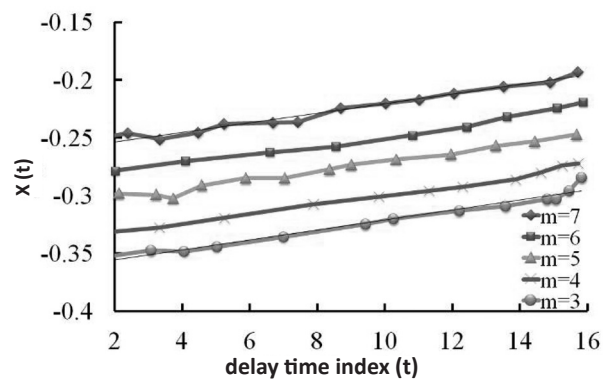


Figure 14: Maximum Lyapunov index of crack tip circumferential stress as a function of delay time.

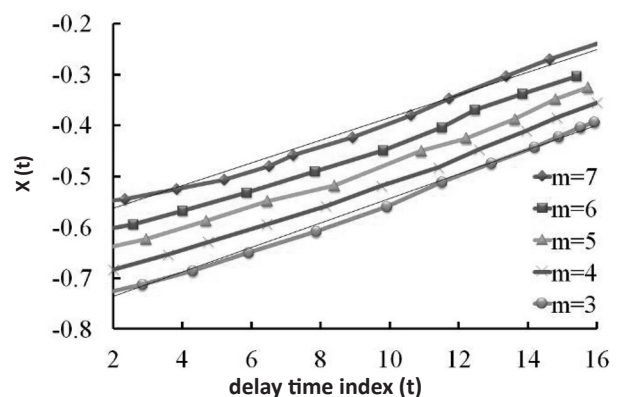


Figure 15: maximum Lyapunov index of crack tip axial stress as a function of delay time.

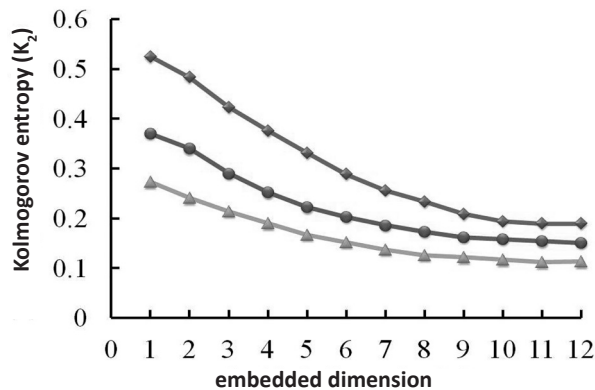


Figure 16: K_2 as a function of m .

Calculation of Kolmogorov Entropy

Using the calculation results of correlation dimension, Kolmogorov entropy can be estimated by Equation 36. Figure 16 shows K_2 variations as a function of embedded dimension (m), in which from top to bottom are micro-crack amount time series, crack tip circumferential stress time series, and crack tip axial stress time series.

As it is shown in Figure 16, a common characteristic of Kolmogorov entropy in three time series is that K_2 gradually decreases and intends to be stable with increasing embedded dimension m . The stable value K_2 needed are 0.179, 0.153, and 0.112, respectively in micro-crack amount time series, crack tip circumferential stress time series, and crack tip axial stress time series.

Calculation and Analysis of 10 Pairs of Experiments

According to the results obtained, we continue to study coal-rock fracturing crack evolution by carrying out 10 experiments and observing the trend Kolmogorov entropy changed with characteristic indicators; the results are shown in Figure 17.

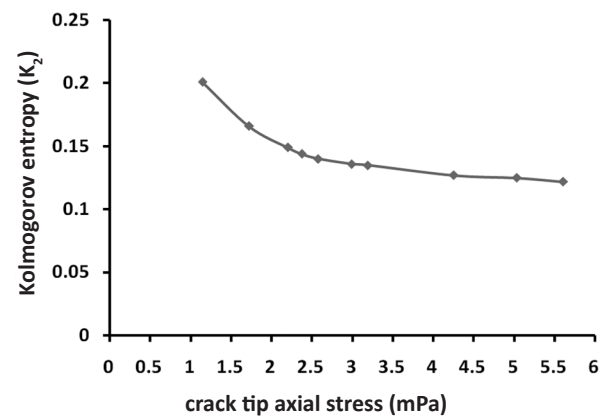
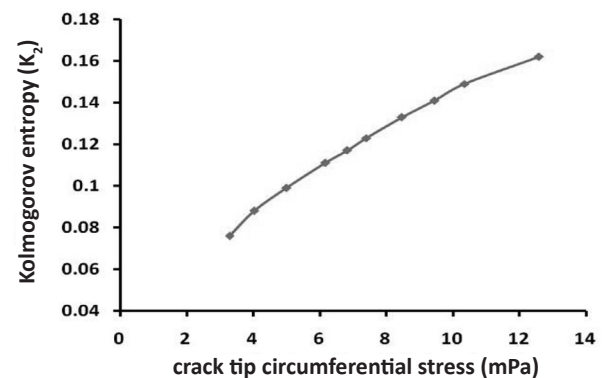
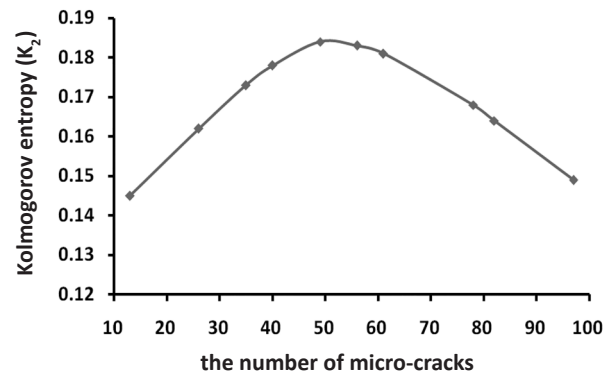


Figure 17: Kolmogorov entropy as a function of characteristic indicators.

It is concluded from the analyses and calculations mentioned above that the correlation dimensions are fractional values, and the maximum Lyapunov index and Kolmogorov entropy are all greater than zero. All these factors meet the conditions of chaos existence, which indicates that crack evolution has a chaotic characteristic. The calculation results provide scientific basis for the evolution and instability of coal-rock fracturing cracks based on the chaos theory.

CONCLUSION

This paper deals with coal-rock fracturing crack evolution based on chaos theory, which is a cutting-edge topic both at home and abroad, and has a great academic value and theoretical significance. The results obtained are as follows:

1. In this paper, based on the model of micro crack evolution developed and by considering the dynamic and static process of fracturing, we derived the expressions of the mechanical distribution characteristics and the numbers of crack evolution of the crack tip.
2. Based on the chaos theory of nonlinear dynamics method, we discussed the evolution behavior of cracks, micro crack amounts, and circumferential stress, and axial stress at crack tip of coal-rock fracturing crack evolution process were used as the characteristic indicators. Correlation dimension, Lyapunov index, and Kolmogorov entropy were introduced as the chaos characteristic quantities. Through the results of 10 experiments, the correlation dimensions were obtained to be fractional values, and the maximum Lyapunov index and Kolmogorov entropy were all greater than zero; these results indicated that coal-rock fracturing crack evolution and the crack network formation process had chaos characteristics.
3. Based on the judgment of chaos, the larger the Kolmogorov entropy is, the higher the crack evolution degree is, and the more complicated the crack system becomes. We calculated the Kolmogorov entropy of 10 experimental data, and the results showed that as the radial stress increased, the Kolmogorov entropy decreased, and a lower degree of chaos was obtained; by increasing the circumferential stress the Kolmogorov entropy increased, and a higher degree of chaos was achieved; as the number of micro cracks evolution increased, the Kolmogorov entropy rose up to a

critical value, and it then dropped, which meant that the chaotic degree increased before they were reduced. All in all, in the process of fracturing, large main cracks are tended to form when radial stress increases, while bifurcation micro cracks are created when circumferential stress increases. At the beginning, the crack system becomes more complicated with cracks evolution, and when the evolution reach a degree, cracks are combined with each other, and the crack system becomes steady.

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