

Performance Model for Vertical Wells with Multi-stage Horizontal Hydraulic Fractures in Water Flooded Multilayer Reservoirs

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ABSTRACT

For the characteristics of horizontal fractures in shallow low-permeability oil layers after hydraulic fracturing in multilayer reservoirs, horizontal fractures are taken equivalent to an elliptical cylinder with the reservoir thickness using the equivalent permeability model; then, upon the elliptic seepage theory, the seepage field which has led by a vertical well with horizontal fractures is divided into two parts: (1) radial flow from the external formation to the equivalent area of horizontal fracture and (2) elliptic flow in the equivalent area of horizontal fracture. The loss of pressure caused by threshold pressure gradient, material balance in the reservoir, and multi-well pressure superposition principle are synthesized to calculate the performance. Finally, separate-layer multi-stage horizontal fractured well performance is deduced by summing the performance of high-permeability oil layers and fractured thin low-permeability oil layers. Low-permeability thin oil layers in Xing Shu-Gang oilfield are taken as practical cases, and the well space limits and economic reservoir thickness limits are calculated by the performance model; the relationship among recovery and productivity intensity, and the ratio of thin low-permeability oil layers thickness to the total thickness are also discussed.

Keywords: Multilayer Reservoir, Low-permeability Thin Oil Layer, Horizontal Fracture, Five-spot Waterflood Pattern, Performance

INTRODUCTION

Low-permeability reservoirs have become main potentials of multi-layer heterogeneous reservoirs, which have reached extreme high water cut. Hydraulic fracture is used frequently to stimulate low-permeability reservoirs, including two types of fractures: (1) horizontal fractures perpendicular to wellbore and (2) vertical fractures parallel to

wellbore. Horizontal fractures are easier to be created in shallow formations, whose vertical stress may be less than horizontal stress [1-3]. Multi-stage fracturing (MSF) has been applied widely to multilayer reservoirs, and horizontal fractures may be generated in shallow oil layers along the vertical wellbore. It is an all-in-one technique for perforating, fracturing, isolating a

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zone, and moving up the wellbore to repeat this process at next desired productive interval [4]. Many authors have studied the performance of vertical wells with vertical fractures [5-7]. However, the performance model of vertical wells with horizontal fractures has been investigated occasionally, and most studies focus on numerical solutions. W. Sung (1987) compared the performance of producers with vertical and horizontal fractures by numerical reservoir simulation [8], and Peter Valko (1998) studied the transient behavior of finite conductivity horizontal fractures [9]; Lasen Leif (2011) studied the pressure in multilayer reservoirs influenced by horizontal fractures [10]. However, few studies have been devoted to an analytical solution for the productivity of vertical wells with horizontal fractures considering oil-water two-phase flow in regular waterflooding patterns. In this paper, an accurate productivity model for vertical wells with horizontal fractures is proposed in conjunction with threshold pressure, material balance, and pressure superposition, for the sake of making better engineering decisions about the development of the reservoirs with complex hydraulic fracturing.

Flow Field Analysis of a Vertical Well with Horizontal Fractures

Model Assumptions

Figure 1a shows the separate-layer multi-stage horizontal fractures which have been studied in this paper, and the following assumptions are made to develop the model.

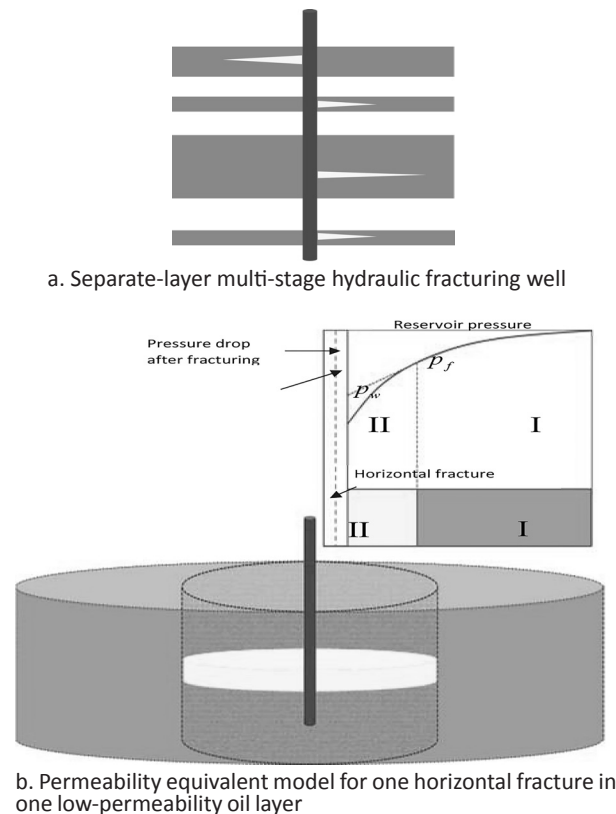


Figure 1: A Separate-layer multi-stage hydraulic fracturing well and a permeability equivalent model.

1. A horizontal fracture is regarded as an elliptical cylinder with a rectangular cross-section, and its height is assumed to be equal with the average fracture width; the fracture is also symmetric around the well.
2. Interlayers grow steadily between every two oil layers, so the vertical flow from upper or lower oil layers to fractures should be ignored.
3. The flow in the reservoir is assumed to be two-phase flow with incompressible fluid of constant viscosity, and capillary force and gravity are negligible.
4. For the sake of simplicity, flow field caused by a horizontal fracture could be equivalent to the one caused by one much larger elliptical cylinder with a height equal to the oil layer thickness, and the larger elliptical cylinder permeability could be calculated by the equivalent permeability model [11]; its shape and pressure distribution are shown in Figure 1b.

5. The flow in the formation with a horizontal fractured well can be divided into two parts: (1) radial flow in the outer zone and (2) elliptical flow in the inner zone. The outer zone is the external formation away from horizontal fracture, and the inner zone, the permeability of which is calculated by a weighted average method, is the equivalent zone for horizontal fractures.

MATHEMATICAL PROCEDURES

Radial Flow in Outer Zone

As Figure 2a shows, the isopiestic lines are a set of confocal elliptical closed lines in the outer zone. These isopiestic lines are more similar to the elliptical boundary when closer to the horizontal fractures, and they change from ellipses to circles gradually when they are away from the fracture.

Liu Yuetian (2000) transformed the radial flow in elliptical surface to parallel flow in a new surface by conformal transformation [12]. According to the transformation, the pressure in the outer zone can be expressed as [12]:

$$p = p_f + C_{SI} \frac{\mu_o B_o Q_{po}}{2\pi K \bar{K}_{ro} h} \ln \frac{a+b}{a_f + b_f} \quad (1)$$

where, p is the pressure in seepage field; p_f is the pressure of the fracture boundary; C_{SI} represents the conversion coefficient of two different units, and μ_o is the viscosity of oil underground; B_o is the oil volume factor, and Q_{po} is the oil production; K is reservoir permeability, and $\bar{K}_{ro}$ stands for the average oil relative permeability; h is reservoir thickness, and a and b are the half long axis and the minor axis of isopiestic lines respectively; a_f and b_f stand for the half long axis and the minor axis of the horizontal elliptical fracture respectively.

Elliptical Flow in Inner Zone

The seepage field from a source well to two parallel

linear boundaries is studied by assuming two infinite linear constant pressure boundaries, which are $2d$ apart and their pressures are both p_e . The source well is located at the origin of coordinates, and these isopiestic lines in seepage field are similar to a series of coaxial elliptical. Assuming that the pressures on the long axis endpoint and the minor axis endpoint of both outer boundaries of horizontal fracture and wellbore are equal (which means $p_{af} = p_{bf} = p_f$ and $p_{aw} = p_{bw} = p_w$), according to pressure superposition principles, the pressure of the horizontal fracture equivalent area can be calculated by [12]:

$$p = p_f + C_{SI} \frac{\mu_o B_o Q_{po}}{2\pi K_f^* \bar{K}_{ro} w_f} \ln \left(\tan \frac{\pi b_f}{4d^*} \right) - C_{SI} \frac{\mu_o B_o Q_{po}}{2\pi K_f^* \bar{K}_{ro} w_f} \ln \left(\frac{ch \frac{\pi x}{2d^*} - \cos \frac{\pi y}{2d^*}}{ch \frac{\pi x}{2d^*} + \cos \frac{\pi y}{2d^*}} \right) \quad (2)$$

where, K_f^* is the equivalent permeability in the inner zone, and it could be calculated by $K_f^* = \frac{K_f w_f + K_l}{h}$; d^* is the parameter determined by the long axis and minor axis of fracture and wellbore: $\tan \frac{\pi b_f}{4d^*} = \tanh \frac{\pi a_f}{4d^*}$, $\tan \frac{\pi b_w}{4d^*} = \tanh \frac{\pi a_w}{4d^*}$ x , y are x-coordinate and y-coordinate of an arbitrary point in flow field, and w_f denotes the average fracture width.

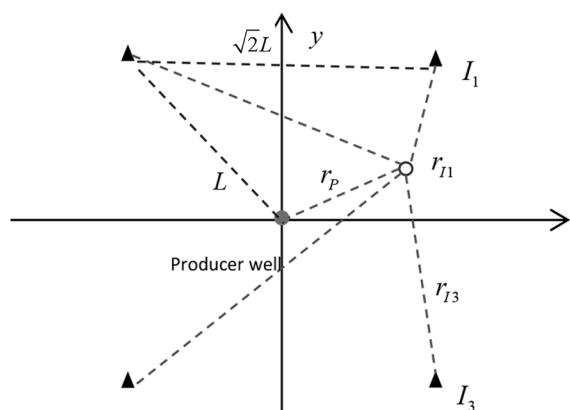
Pressure Loss Caused by Threshold Pressure Gradient in a Five-spot Waterflooding Pattern

The existence of threshold pressure gradient (TPG) has provoked much controversy in previous investigations [13-14]. Actually, low-permeability oil layers are developed more difficultly compared to high-permeability formations in waterflooded multilayer reservoirs, and threshold pressure gradient undoubtedly provides a quit reasonable interpretation for this difficulty. According to the pressure superposition principle, reservoir pressure with TPG

where, r_w , L , α , and β represent wellbore radius, the distance between a producer well and an injector well, the ratio of water injector well number to producer well number, and a practical parameter respectively.

Performance of a Producer with One Horizontal Fracture in a Five-spot Pattern

For a water injector well, the main pressure drop is near its wellbore, so the influence of distal formation with horizontal fracture can be negligible. According to pressure superposition principles, the pressure drop caused by four water injector wells and one producer well (the five-spot waterflooding pattern) is superimposed respectively in the two zones, and then seepage field pressure can be calculated respectively; the pressure loss caused by TPG is not considered when the pressure in these two zones are discussed because the pressure loss is not influenced by different flow patterns; thus it is assumed to be equal with the one calculated in the regular five-spot pattern above.



1. Pressure in the outer zone

$$\begin{aligned}
p &= p_e + \Delta p_I + \Delta p_P \\
&= p_w - C_{SI} \frac{\mu_o \underline{Q}_{P_o} B_o}{2\pi K_f^* \bar{K}_{ro} h} \ln \left(\tan \frac{\pi b_f}{4d^*} \right) \\
&\quad + C_{SI} \frac{\mu_o \underline{Q}_{P_o} B_o}{2\pi K_f^* \bar{K}_{ro} h} \ln \left(\frac{ch \frac{\pi x_w}{2d^*} - \cos \frac{\pi y_w}{2d^*}}{ch \frac{\pi x_w}{2d^*} + \cos \frac{\pi y_w}{2d^*}} \right) \\
&\quad + C_{SI} \frac{\mu_o \underline{Q}_{P_o} B_o}{2\pi K \bar{K}_{ro} h} \ln \frac{a+b}{a_f+b_f} \\
&\quad + \sum_{m=1}^n \left(p_I - C_{SI} \frac{\mu_w \underline{Q}_{Iw} B_w}{2\pi K \bar{K}_{rw} h} \ln \frac{r_{Im}}{r_w} - p_e \right) \quad (5)
\end{aligned}$$

where, p_e is the initial reservoir pressure, and Δp_I denotes the pressure drop caused by water injector well; Δp_p is the pressure drop caused by producer

well, and $\bar{K}_{rw}$ represents the average water relative permeability.

2. Pressure in the inner zone

$$p = p_w + C_{SI} \frac{\mu_o Q_{Po} B_o}{2\pi K_f \bar{K}_{ro} h} \cdot \ln \left[\left(\frac{ch \frac{\pi x_w}{2d^*} - \cos \frac{\pi y_w}{2d^*}}{ch \frac{\pi x_w}{2d^*} + \cos \frac{\pi y_w}{2d^*}} \right) \div \left(\frac{ch \frac{\pi x}{2d^*} - \cos \frac{\pi y}{2d^*}}{ch \frac{\pi x}{2d^*} + \cos \frac{\pi y}{2d^*}} \right) \right] + \sum_{m=1}^n \left(p_I - C_{SI} \frac{\mu_w Q_{fw} B_w}{2\pi K \bar{K}_{rw} h} \ln \frac{r_{lm}}{r_w} - p_e \right) \quad (6)$$

Hence injection-production pressure difference can be given as:

$$p_I - p_w = -C_{SI} \frac{\mu_o Q_{Po} B_o}{2\pi K_f \bar{K}_{ro} h} \ln \left(\tan \frac{\pi b_f}{4d^*} \right) + C_{SI} \frac{\mu_o Q_{Po} B_o}{2\pi K_f \bar{K}_{ro} h} \ln \left(\frac{ch \frac{\pi x_w}{2d^*} - \cos \frac{\pi y_w}{2d^*}}{ch \frac{\pi x_w}{2d^*} + \cos \frac{\pi y_w}{2d^*}} \right) + C_{SI} \frac{\mu_w Q_{fw} B_w}{2\pi K K_{w \max} h} \ln \frac{(\sqrt{2}L - r_w)^2 (2L - r_w) r_w}{(L - r_w)^4} + C_{SI} \frac{\mu_o Q_{Po} B_o}{2\pi K \bar{K}_{ro} h} \ln \frac{a_I + b_I}{a_f + b_f} \quad (7)$$

The oil production of a producer well with one horizontal fracture in the five-spot waterflooding pattern can be calculated by considering the pressure loss caused by TPG:

$$Q_{Po} = \frac{p_I - p_w - \beta \lambda \left(L - r_w - \frac{a_f + b_f}{2} \right)}{\left[C_{SI} \frac{\mu_o B_o}{2\pi K \bar{K}_{ro} h} \ln \frac{a_I + b_I}{a_f + b_f} - C_{SI} \frac{\mu_o B_o}{2\pi K_f \bar{K}_{ro} h} \ln \left(\tan \frac{\pi b_f}{4d^*} \right) + C_{SI} \frac{\mu_o B_o}{2\pi K_f \bar{K}_{ro} h} \ln \left(\frac{ch \frac{\pi x_w}{2d^*} - \cos \frac{\pi y_w}{2d^*}}{ch \frac{\pi x_w}{2d^*} + \cos \frac{\pi y_w}{2d^*}} \right) + C_{SI} \alpha \left(1 + \frac{\bar{K}_{rw} \mu_o B_o}{\bar{K}_{ro} \mu_w B_w} \right) \frac{\mu_w B_w}{2\pi K \bar{K}_{rw} h} \cdot \ln \frac{(L - r_w)^4}{(\sqrt{2}L - r_w)^2 (2L - r_w) r_w} \right]} \quad (8)$$

Similarly, water production is given by:

$$Q_{Pw} = \frac{p_I - p_w - \beta \lambda \left(L - r_w - \frac{a_f + b_f}{2} \right)}{\left[C_{SI} \frac{\mu_o B_o}{2\pi K \bar{K}_{ro} h} \ln \frac{a_I + b_I}{a_f + b_f} - C_{SI} \frac{\mu_o B_o}{2\pi K_f \bar{K}_{ro} h} \ln \left(\tan \frac{\pi b_f}{4d^*} \right) + C_{SI} \frac{\mu_o B_o}{2\pi K_f \bar{K}_{ro} h} \ln \left(\frac{ch \frac{\pi x_w}{2d^*} - \cos \frac{\pi y_w}{2d^*}}{ch \frac{\pi x_w}{2d^*} + \cos \frac{\pi y_w}{2d^*}} \right) + C_{SI} \alpha \left(1 + \frac{\bar{K}_{rw} \mu_o B_o}{\bar{K}_{ro} \mu_w B_w} \right) \frac{\mu_w B_w}{2\pi K \bar{K}_{rw} h} \cdot \ln \frac{(L - r_w)^4}{(\sqrt{2}L - r_w)^2 (2L - r_w) r_w} \right]} \quad (9)$$

Performance of the Five-spot Pattern after Separate-layer Multi-stage Fracturing

The productivity of every low-permeability oil layer can be summed to calculate the whole productivity of vertical well with separate-layer multi-stage hydraulic fracturing using Equations 9-10. There is no need to take into account the influence of TPG for high-permeability oil layers in multilayer reservoirs. Assuming that all the low-permeability oil layers create horizontal fractures after separate-layer multi-stage hydraulic fracturing, the oil production of five-spot pattern after separate-layer multi-stage hydraulic fracturing can be calculated as defined below:

$$Q_{Po}^{Total} = \sum_{i=1}^c Q_{Poi} + \sum_{j=1}^d Q_{Poj} \quad (10)$$

where c is the sum of high-permeability oil layers, and d is the sum of low-permeability oil layers; i and j are number marks.

In a similar way, the water production of a producer well can be calculated as:

$$Q_{Pw}^{Total} = \sum_{i=1}^c Q_{Pwi} + \sum_{j=1}^d Q_{Pwj} \quad (11)$$

Thus the water content reads:

$$f_w = \frac{Q_{Pw}^{all}}{Q_{Pw}^{all} + Q_{Po}^{all}} \times 100\% \quad (12)$$

On the basis of material balance, average oil saturation is calculated in different periods by assuming that liquid volume produced in a time step is equal to injection water volume, so oil saturation difference is equal to the ratio of oil production volume to reservoir pore volume; then the oil saturation in a new time step is calculated, and it is used to choose the oil and water relative permeability upon oil-water relative permeability curve. Oil production and water content are calculated for the next time step.

$$\Delta \bar{S}_w = -\Delta \bar{S}_o = \frac{Q_{Po} B_o \Delta t}{V_\phi} \quad (13)$$

Model Validation

According to the test data of producer X5-41-S737 in the thin low-permeability oil layer of Xing No.6 block which is located at Daqing oilfield, the results which are calculated by the performance model are validated. As shown in Figure 3, daily oil production has been calculated by our model and the test data of X5-41-S737 only differ slightly; decline trends are basic coincidence, so the performance model is reliable, and it could be used to design development projects for thin low-permeability oil layers in Daqing oilfield.

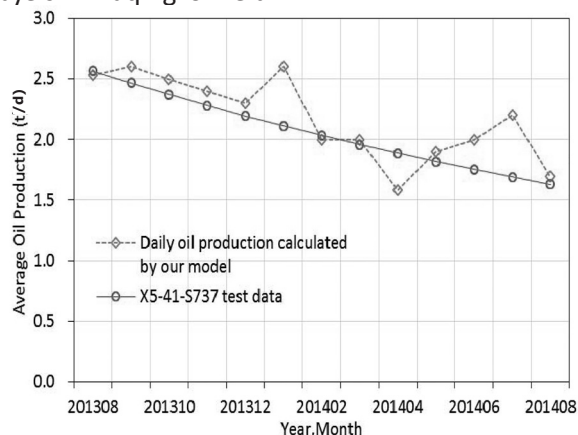


Figure 3: Daily oil production calculated by our model and the test data of X5-41-S737.

CASE STUDY

The Xing Shu-Gang oilfield is located in Daqing, China. It belongs to shallow sandstone multilayer reservoirs. Table 1 shows the basic parameters of the reservoirs, fractures, and fluids. Figure 4 displays the oil-water relative permeability curve for high-permeability oil layer and low-permeability oil layer respectively. The productivity of a producer well with multi-stage horizontal fractures is calculated, and the minimum well space, economic oil layer thickness, recovery, and productivity intensity are discussed.

Table 1: Physical parameters of reservoirs, fractures, and fluids.

Basic Parameters	High-permeability Oil Layers	Low-permeability Oil Layers
Porosity	0.25	0.22
Permeability (mD)	100~1000	5~10
Average half axis (m)	-	30
Fracture width (m)	-	0.003
Dimensionless fracture conductivity	-	0.3
Injection-production pressure difference (MPa)	20	20
Initial oil saturation	0.72	0.5
Oil volume factor	1.2	1.2
TPG (MPa/m)	0	0.025
Fluid viscosity(MPa.s)	7.8	7.8

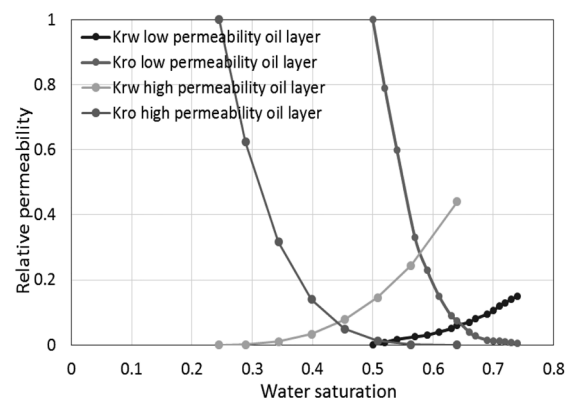
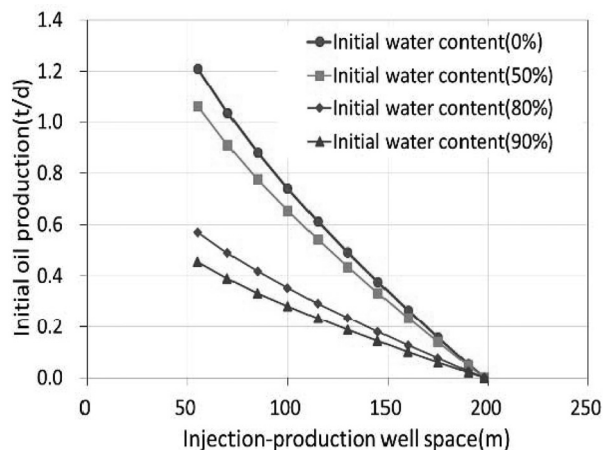


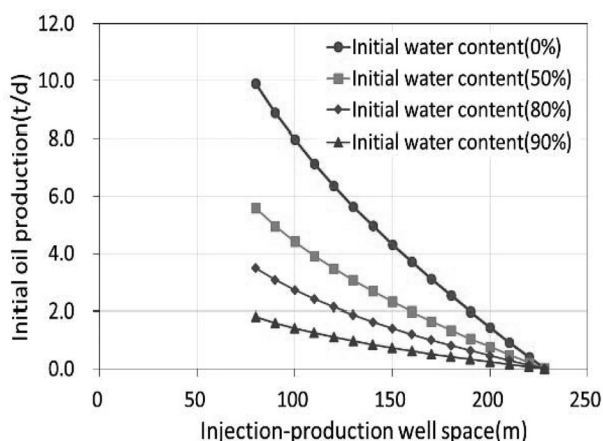
Figure 4: Relative permeability curves for both high-permeability and low-permeability oil layers.

Minimum Well Space and Economic Oil Layer Thickness

The minimum well space is the injection-production well space which producer well has the minimum oil production by waterflooding in the five-spot pattern. Economic oil layer thickness is the thickness of oil layer which could have economic production. Assuming that a vertical producer well only perforates some low-permeability oil layers, then the initial oil production under different initial water cut, well space, and oil layer thickness after hydraulic fracturing are calculated using Equation 10. Figures 5 and 6 show the results of initial oil production.

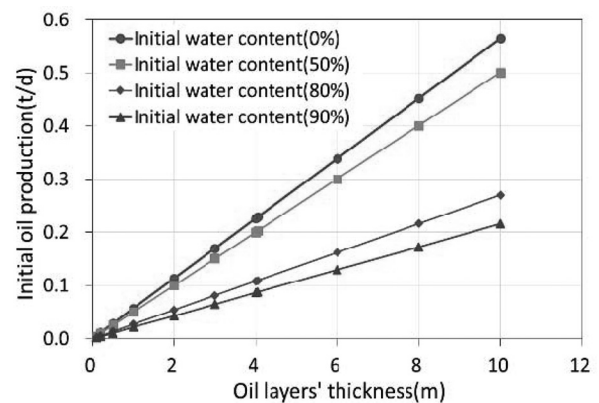


a. Initial oil production changing with injection-production well space in 6m oil layers without fracturing.

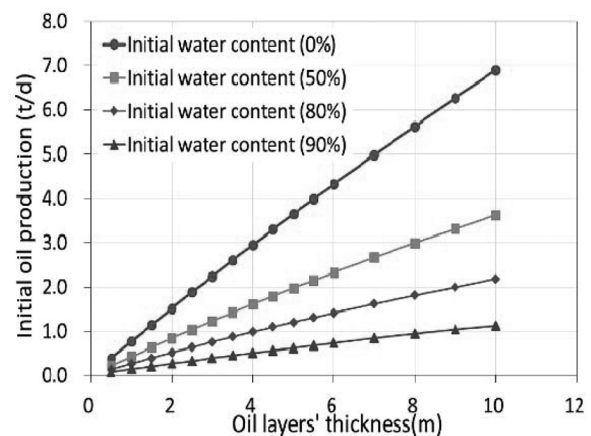


b. Initial oil production changing with oil layers thickness under 150m well space with fracturing.

Figure 5: The relationship between oil production and well space



a. Initial oil production changing with oil layers thickness under 150m well space without fracturing.



b. Initial oil production changing with oil layers thickness under 150m well space with fracturing.

Figure 6: The relationship between initial oil production and oil layers thickness.

As Figure 5 shows, the minimum well space is 198 m before hydraulic fracturing, and it extends to 228 m after hydraulic fracturing, so it is lengthened by 30 m because of horizontal fracturing. When the oil layer thickness is 6 m, the production-injection well space is 150 m, and the initial oil production will be less than 0.4 t/d under any initial water content; thus it has not reached the economic productivity of Xing Shu-Gang oil field (1.5 t/d each producer); however, oil production increases to more than 2 t/d after hydraulic fracturing with an initial water content less than 50%.

As shown in Figure 6, the initial oil production enhances with increasing oil layer thickness. Even if oil layer thickness increases to 10 m, the initial oil production will be as low as 0.57 t/d without fracturing under an initial water content of 0%; thus it has not reached the economic oil production. After hydraulic fracturing, the initial oil production increases to 6.9 t/d with a water content of 0%; however, when the oil-layer thickness decreases to 2 m, the initial oil production is generally less than 1.5 t/d, which causes the oil layer to lose economic recoverable values.

The Relationship between Recovery and Ratio of Low-permeability Oil Layers Thickness to the Total Thickness

Assuming that the vertical producer well perforates both high-permeability and low-permeability oil layers, the relationship between recovery and ratio of low-permeability oil layers thickness to total thickness is analyzed as follows. Figure 7 shows that with increasing the ratio, its recovery first drops, levels off, and then increases rapidly. This may be because of the fact that oil production is lower, and the water content increases more slowly from low-permeability oil layers than from high-permeability oil layers. When the ratio is low, high-permeability oil layers would play a more important role in providing oil production. In addition to, when the ratio is low, injection water flows mainly through high-permeability oil layers, and the producer well reaches extreme water content (98%) suddenly. However, the total recovery of the multilayer reservoir is low too because the recovery of these low-permeability oil layers maintains quite low. When the ratio is high, low-permeability oil layers would play a more important

role in oil production. Although some injection water flows through high-permeability oil layers, which does not cause the producer well to reach the extreme water content (98%) suddenly, the total recovery enhances.

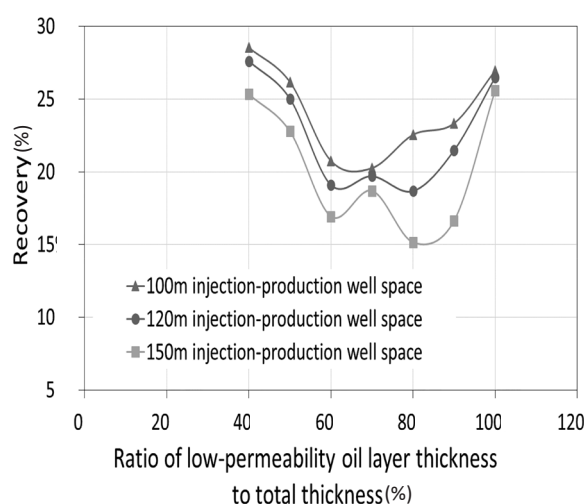


Figure 7: The relationship between recovery and ratio of low-permeability oil layer thickness to total thickness.

The Relationship between Productivity Intensity and Ratio of Low-permeability Oil Layers Thickness to Total Thickness

Assuming that the vertical producer well perforates both high-permeability and low-permeability oil layers, the relationship between productivity intensity and the ratio of low-permeability oil layers thickness to total thickness is analyzed as follows. Figure 8 shows that productivity intensity gradually decreases with increasing the ratio, but the productivity intensity gradually decreases with increasing well space. That is because the productivity intensity of low-permeability oil layer is weaker than the high-permeability oil layer; also, if the ratio becomes higher, the productivity intensity of the whole multilayer reservoir becomes weaker. With increasing well space, the supply of water injector well becomes weaker, and then the productivity intensity of the whole multilayer reservoir lessens.

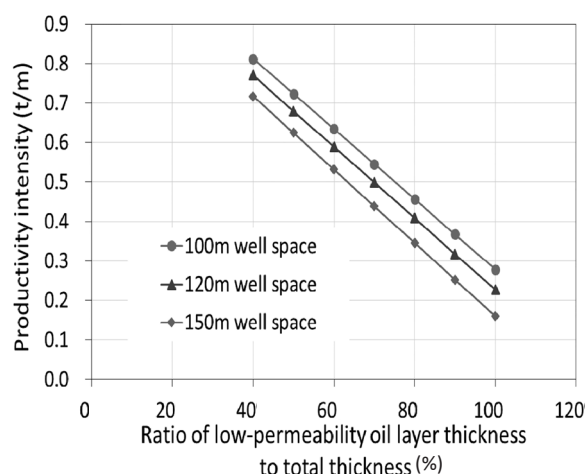


Figure 8: The relationship between productivity intensity and the ratio of low-permeability oil layer thickness to total thickness.

CONCLUSIONS

1. The performance model of vertical well with separate-layer multi-stage horizontal fractures is derived by considering the loss of pressure caused by TPG, material balance, and pressure superposition.
2. A high-precision analytical solution method for this performance model is presented by assuming the average decrement of water saturation in the reservoir, so the evaluation process of the performance has been simplified.
3. The minimum well space and economic oil layer thickness are calculated by the performance model for the multilayer reservoir in Xing Shu-Gang oilfield. When the total low-permeability oil layer thickness is less than 10 m, the initial oil production without fracturing is less than 1.5 t/d, so it will lose the recoverable value. When the well space is less than 150 m with initial water content less than 50%, the initial oil production after fracturing is more than 2 t/d, which has the recoverable value. When the total thickness of low-permeability oil layer decreases to 2 m, the initial oil production after fracturing is less than 1.5 t/d, and it will lose the recoverable value.

4. The relationship among recovery, productivity intensity, and the thickness ratio of low-permeability oil layers to high-permeability oil layers is studied. With increasing the thickness of low-permeability oil layer, first recovery first decreases, second levels off, and finally increases rapidly; however, the productivity intensity gradually drops.

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