The Analytic Technique and Experimental Research Methods of Post-buckling about Slender Rod Strings in Wellbore

Yue Qian Bei, Liu Ju-bao*, and Fu Mao-qing

College Mechanical Science Engineering, Northeast Petroleum University, Hei Long-Jiang, Daqing,China

ABSTRACT

The buckling behavior of rod strings in wellbore is one of the key issues in petroleum engineering. The slender rod strings in vertical wellbore were selected as research objects. Based on the energy method, the critical load formulas of sinusoidal and helical buckling were derived for the string with the bottom of the wellbore pressure. According to the sinusoidal and helical buckling's geometry of the pressurized column, the contact friction state of post-buckling between the rod strings and the wellbore was considered. This paper adopted Lagrange multiplier method to describe sliding displacement boundary conditions of wellbore and introduced gravitational potential energy and friction resistance dissipated energy in the energy method. The contact forces and friction resistances between the rod string and wellbore were derived in vertical wells. The post buckling experimental apparatus was also developed in this paper. The sinusoidal and helical buckling critical loads and friction resistances were derived under different loads. The experimental results are consistent with the theoretical results. Therefore, this article would provide an effective method for buckling column such as drilling and coiled tubing.

Keywords: Sinusoidal Buckling, Helical Buckling, Friction Resistance Dissipated Energy, Contact Force, Friction Resistance

INTRODUCTION

The buckling problem of slender rod string within the wellbore has many adverse effects for petroleum engineering jobs, particularly in drilling and workover jobs. Because of the friction forces surge after bucking the weight of rod string cannot be added to the drill bit, which will cause the failure of drill and workover jobs. Thus, it will limit the application of coiled tubing and the drill depth for large displacement wells [1, 2]. The buckling behavior of slender rod string in wellbore has been a hot topic of concern by researchers at home and abroad. There are several typical research methods: the classical differential equation method [3, 4], the finite element method [5, 6], the energy method, and so on, among which the energy method is more used to solve the buckling problems of rod strings. While in traditional energy method the gravitational potential energy as well as friction resistance dissipated energy are not considered for buckling behavior impact [7-9], this paper selects the slender rod strings in vertical wellbore as the research object. Energy method based on slender rod strings later buckling behavior is used to perform research. The formulas of the critical loads, contact forces, and friction resistances for sinusoidal and helical buckling are established for slender rod string in wellbore. The results of this research will contribute to the promotion and

***Corresponding author**

LIU Ju-bao

Email: jslx2000@163.com

Tel: +86 1394 6946485

Fax: +86 0459 6507998

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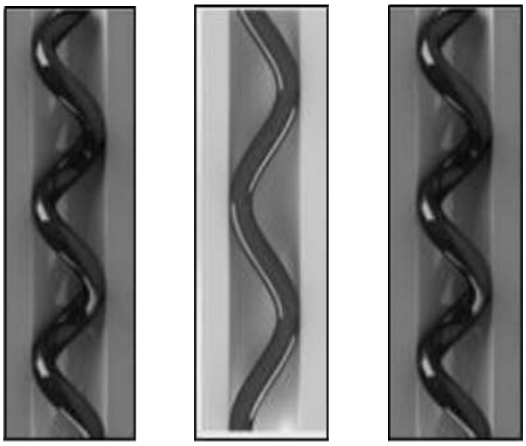
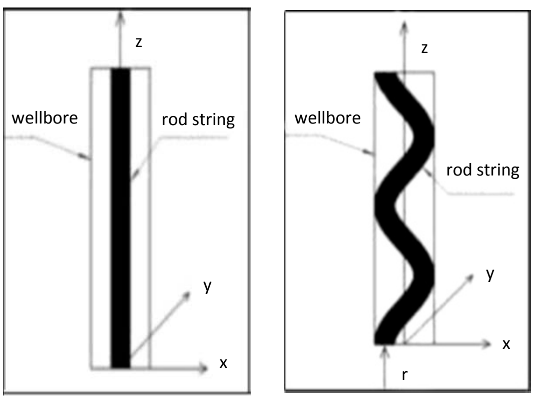
application of coiled tubing for drilling and workover jobs and reach horizontal well technology.

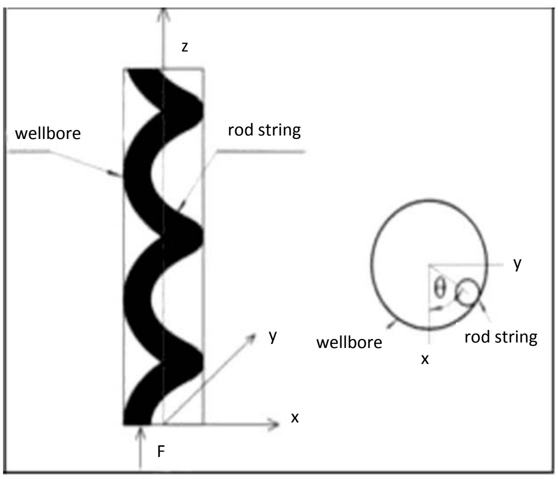


(a) Initial position (b) sine (c) helix

Figure 1: The buckling shape of slender rod string in vertical wells.



(a) Initial position (b) sine



(c) Helix

Figure 2: The geometric relationships of buckling for a slender rod string in vertical wells.

The Geometric Relationships of Sinusoidal and Helical Buckling about Slender Rod in Wellbore

In vertical wells, the initial position of the slender rod is shown in Figure 1 (a). The destabilization of the string occurs under axial pressure and fluid pressure of the bottom. The wellbore will lose axial strength without constraints, and, because of cylinder constraints, it will make the rod string generate sinusoidal buckling along cylinder with the increase of the axial pressure, as shown in Figure 1 (b). When the axial load continues to increase, the lateral deformation of the rod string sinusoidal buckling configuration will increase. When the load reaches a certain critical value, the buckling configuration of the string will become helical, and it maintains continuous contact with the inner wall of cylinder, as depicted in Figure 1 (c).

According to the geometry of sinusoidal and helical buckling occurred by a slender rod string (shown in Figure 2), the geometric relationships were established in Equations 1 and 2 respectively.

$$\begin{cases} x = r \sin \dfrac{2\pi z}{p} \\ y = 0 \\ z = z \end{cases} \quad (1)$$

where *x* and *y* stand for the transverse displacement of the column in the coordinate system; *z* is the axial displacement; *p* is the wavelength of sinusoidal, and *r* represents the annular clearance.

$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \\ \theta = \dfrac{2\pi z}{p} \end{cases} \quad (2)$$

where *p* is the screw pitch of helical buckling, and θ is the angle from the initial position of strings; the rest letters have the same meaning as in Equation 1.

The following assumptions are used: (1) the boundary conditions of slender rod string do not affect the relationship between force and wavelength; (2) the deformation of the string is small; (3) the cross section of the wellbore is circular and rigid body; (4) the effect of fluid action is not considered; and (5) the dynamic effects are neglected.

The Energy Method for Sinusoidal Buckling Critical Load of Slender Rod Column in Vertical Well

When the slender rod string buckling occurs, the total potential energy includes the bending deformation energy U_b, the power generated by bottom hole pressure Ω_F, and the gravitational potential energy W; the related correlation is shown as follows:

$$V = U_b - \Omega_F + W \tag{3}$$

Assuming $d\lambda$ is the compression displacement generated by bending deformation occurred by dz part of the rod string, the expression is given by:

$$d\lambda = dz - dz' = dz - \sqrt{dz^2 - (dy^2 + dx^2)} \approx \frac{1}{2}\left(\left(\frac{dx}{dz}\right)^2 + \left(\frac{dy}{dz}\right)^2\right)dz \tag{4}$$

Thus the deformation of dz section of the rod string is:

$$d\lambda = \frac{1}{2}\left(x'^2 + y'^2\right)dz \tag{5}$$

For vertical wells, because of y=0, its deformation for dz segment can be expressed as:

$$d\lambda = \frac{1}{2}x'^2 dz \tag{6}$$

According to the assumption of small deformation, the bending deformation energy U_b of the rod strings is defined by:

$$U_b = \frac{1}{2}\int_0^L EIx''^2 dz \tag{7}$$

where E is the elastic modulus of the rod string; I represents the moment of inertia, and L stands for the total length.

The power of bottom hole pressure is:

$$\Omega_F = \frac{1}{2}\int_0^L Fx'^2 dz \tag{8}$$

The gravitational potential energy W is:

$$W = \frac{1}{2}\int_0^L wz\,x'^2 dz \tag{9}$$

The first and the second derivatives of x with respect to z are substituted in the expression of U_b, Ω_F, W and through a series of mathematical integration are shown as follows:

$$\begin{cases} U_b = \dfrac{4\pi^4 r^2 EIL}{p^4} - \dfrac{\pi^3 r^2 EI}{p^3}\sin\dfrac{4\pi L}{p} \\ \Omega_F = F\left(\dfrac{\pi^2 r^2 L}{p^2} + \dfrac{\pi r^2}{4p}\sin\dfrac{4\pi L}{p}\right) \\ W = \dfrac{r^2 w}{2}\left(\dfrac{\pi^2 L^2}{p^2} + \dfrac{1}{4}\sin\dfrac{4\pi L}{p} + \dfrac{1}{8}\cos\dfrac{4\pi L}{p} - \dfrac{1}{8}\right) \end{cases} \tag{10}$$

Substitute Equation 10 into Equation 3, the total potential energy of vertical wells is obtained by:

$$V = \left(\frac{4\pi^4 r^2 EIL}{p^4} - \frac{\pi^3 r^2 EI}{p^3}\sin\frac{4\pi L}{p}\right) - F\left(\frac{\pi^2 r^2 L}{p^2} + \frac{\pi r^2}{4p}\sin\frac{4\pi L}{p}\right) + \frac{r^2 w}{2}\left(\frac{\pi^2 L^2}{p^2} + \frac{1}{4}\sin\frac{4\pi L}{p} + \frac{1}{8}\cos\frac{4\pi L}{p} - \frac{1}{8}\right) \tag{11}$$

Considering the system energy conservation, assign V=0, so that

$$F_b = \frac{\pi EI}{p^2}\left(\frac{\dfrac{4\pi L}{p} - \sin\dfrac{4\pi L}{p}}{\dfrac{\pi L}{p} + \dfrac{1}{4}\sin\dfrac{4\pi L}{p}}\right) + \frac{\dfrac{w}{2}\left(\dfrac{\pi^2 L^2}{p^2} + \dfrac{1}{4}\sin\dfrac{4\pi L}{p} + \dfrac{1}{8}\cos\dfrac{4\pi L}{p} - \dfrac{1}{8}\right)}{\left(\dfrac{\pi^2 L}{p^2} + \dfrac{\pi}{4p}\sin\dfrac{4\pi L}{p}\right)} \tag{12}$$

$$F=\frac{\pi^2 EI}{L^2}+\frac{wL}{2} \quad (13)$$

Assigning $\frac{\partial F_b}{\partial p}=0$, then one obtain p=2L; substituting p into Equation 12, the critical load of sinusoidal buckling of the rod strings in vertical wells is shown as follows:

$$F=\frac{\pi^2 EI}{L^2} \quad (14)$$

In the total potential energy, without considering the gravitational potential energy, the critical load of sinusoidal buckling is:

If $\frac{\partial^k F_b}{\partial p^k}=0$ (k=1, 2, 3 ...), then one may obtain:

$$p_k=\frac{2}{2k-1}L \quad (15)$$

Substituting p_k into Equation 12, the K-order critical load of sinusoidal buckling is defined by:

$$F_k=\frac{(2k-1)^2\pi^2 EI}{2L^2}+\frac{wL}{2}, \quad \text{of } (k=1, 2, 3 \ldots) \quad (16)$$

Without considering the gravitational potential energy, the K-order critical load of sinusoidal buckling is given by:

$$F_k=\frac{(2k-1)^2\pi^2 EI}{2L^2}, \quad \text{of } (k=1, 2, 3 \ldots) \quad (17)$$

The Energy Method for Helical Buckling Critical Load of Slender Rod Column in Vertical Well

Substituting the first and the second derivatives of θ into Equation 3, the formula of the total potential energy is obtained for slender rod string in helical buckling.

$$V=\frac{EIr^2}{2}\int_0^L\left(\frac{2\pi}{p}\right)^4 dz-\frac{r^2}{4}\int_0^L F_b\left(\frac{2\pi}{p}\right)^2 dz+\frac{r^2}{4}\int_0^L wL\left(\frac{2\pi}{p}\right)^2 dz$$

The above equation is simplified to:

$$V=\frac{EIr^2}{2}\left(\frac{2\pi}{p}\right)^4 L-\frac{r^2}{4}F_b\left(\frac{2\pi}{p}\right)^2 L+\frac{r^2}{4}w\left(\frac{2\pi}{p}\right)^2 L^2 \quad (18)$$

Considering the system energy conservation, the critical load of helical buckling for strings in vertical wells is given by:

$$F_b=2EI\left(\frac{2\pi}{p}\right)^2+\frac{wL}{2} \quad (19)$$

The Analysis of Sinusoidal Buckling Contact Force of Slender Rod String in Wellbore

Adopting Lagrange multiplier method, the contact force and frictional resistance between string and wellbore are analyzed for sinusoidal buckling. Considering the gravitational potential energy and friction resistance dissipated energy, the total potential energy of the system is obtained for strings.

$$V=U_b-\Omega_F+W+\Omega_\lambda-\Omega_{\lambda f} \quad (20)$$

where the expressions of U_b, Ω_F, and W are seen in Equation 10. The Ω_λ is the contact constraints of the wellbore to rod string. The $\Omega_{\lambda f}$ is the friction resistance dissipated energy generated by sinusoidal buckling. The formulas are shown in Equations 21 and 22 respectively.

$$-\Omega_\lambda=\int_0^L \lambda g(x,y)dz \quad (21)$$

where λ is the contact force between string and wellbore per unit length; $g(x,y)=\sqrt{x^2+y^2}-r$ is constraint function.

$$\Omega_{\lambda f}=\frac{1}{2}\int_0^L \lambda f \; z[(x')^2+(y')^2]dz \quad (22)$$

where, f is the friction coefficient between string and the wellbore.

Compared with Equation 3, the total potential energy of system can be expressed as:

$$V_m = \int_0^L f(z,x,x',x'')dz + \int_0^L \lambda g(x,y)dz \qquad (23)$$

where the first term is the sum of potential energy and friction resistance dissipated energy, which can be expressed as:

$$f(z,x,x',x'') = \frac{EI}{2}(x'')^2 - \frac{F}{2}(x')^2 + \frac{wz}{2}(x')^2 - \frac{\lambda fz}{2}(x')^2$$

Taking Equation 23 into a step variation:

$$\delta V_m = \int_0^l [\frac{df}{dx} - \frac{d}{dz}(\frac{df}{dx'}) + \frac{d^2}{dz^2}(\frac{df}{dx''}) + \lambda \frac{\partial g}{\partial x}]\delta x dz \qquad (24)$$

According to the principle of minimum potential energy we have $\delta V_m = 0$, so the governing differential equations of pipe string are expressed as follows:

$$\frac{df}{dx} - \frac{d}{dz}(\frac{df}{dx'}) + \frac{d^2}{dz^2}(\frac{df}{dx''}) + \lambda \frac{\partial g}{\partial x} = 0 \qquad (25)$$

Substitute the derivatives of function *f* with respect to *x* and *z* into Equation 25, one may obtain:

$$\frac{d}{dz}(Fx' - wzx' + \lambda fzx') + \frac{d^2}{dz^2}(EIx'') + \lambda = 0 \qquad (26)$$

Having substituted the first two derivatives of *x* with respect to *z* into the above equation, the formula of contact force per unit length is obtained in vertical wells for strings with sinusoidal buckling.

$$\lambda = \frac{w\cos\left(\frac{2\pi}{p}z\right) + (F - wz)\left(\frac{2\pi}{p}r\right)^2 \sin\left(\frac{2\pi}{p}z\right)}{f\cos\left(\frac{2\pi}{p}z\right) - \left(\frac{2\pi}{p}r\right)^2 fz\sin\left(\frac{2\pi}{p}z\right) + 1} - \frac{EI\left(\frac{2\pi}{p}r\right)^4 \sin\left(\frac{2\pi}{p}z\right)}{f\cos\left(\frac{2\pi}{p}z\right) - \left(\frac{2\pi}{p}r\right)^2 fz\sin\left(\frac{2\pi}{p}z\right) + 1} \qquad (27)$$

In the calculation of contact force with Lagrange multiplier method, without considering gravitational potential energy and friction resistance dissipated energy, the formula of contact force per unit length for columns with sinusoidal buckling in vertical wells is defined by:

$$\lambda = F\left(\frac{2\pi}{p}r\right)^2 \sin\left(\frac{2\pi}{p}z\right) - EI\left(\frac{2\pi}{p}r\right)^4 \cos\left(\frac{2\pi}{p}z\right) \qquad (28)$$

The Analysis of Helical Buckling Contact Force of Slender Rod Column in Wellbore

In the analysis of helical buckling for slender rod string, substituting $\theta = \frac{2\pi z}{p}$、 $\theta' = \frac{2\pi}{p}$, and $\theta'' = 0$ into Equation 27, the formula of contact force per unit length for strings with helical buckling in vertical wells is obtained by:

$$\lambda = \frac{EIr\left[-\left(\frac{2\pi}{p}\right)^4\right] + (F - wz)r\left(\frac{2\pi}{p}\right)^2}{1 - frz\left(\frac{2\pi}{p}\right)^2} \qquad (29)$$

Without considering gravitational potential energy and friction resistance dissipated energy, the formula of contact force per unit length of strings with helical buckling in vertical wells is expressed as follows:

$$\lambda = EIr\left[-\left(\frac{2\pi}{p}\right)^4\right] + Fr\left(\frac{2\pi}{p}\right)^2 \qquad (30)$$

Example Calculation and Experimental Verification

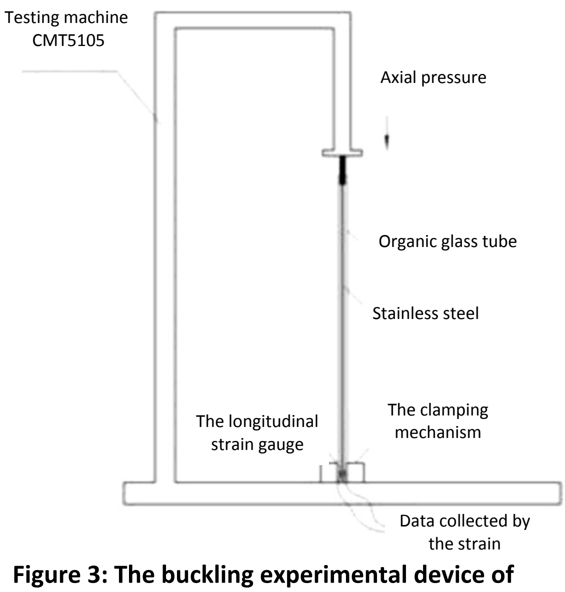


Figure 3: The buckling experimental device of slender string.

In order to verify the accuracy of the critical load and friction resistance for sinusoidal and helical buckling, the bucking indoor tests are designed for slender column within an organic glass tube. In this experiment, the stainless steel tube as the slender rod is used to simulate the pipe column. The size of the stainless steel tube is as follows: outer diameter= 6 mm; thickness= 1 mm; and length= 790 mm. The organic glass tube is used to simulate the wellbore. The size of the organic glass tube is as follows: outer diameter= 30 mm; thickness= 3 mm; and length= 800 mm. An electronic universal tensile testing machine CMT5105 was used to provide axial pressure. A dynamic signal measurement JM5930 was also used to measure the bottom strain of the organic glass tube at different axial pressures. The experimental apparatus is shown in Figure 3.

The Comparison Results of Critical Load with Experimental and Theoretical Calculation of Slender Strings with Buckling

The 1-10 group specimens of stainless steel tube were used to carry out a lot of buckling experiments. The sinusoidal and helical buckling shape occurred is shown in Figure 4.

(a) sine (b) helix

Figure 4: The slender string with helical buckling.

The critical loads of sinusoidal and helical buckling are obtained for the 10 groups of specimens. At the same time, Equations 8 and 13 are used to calculate the critical load. The contrasting results are shown in Tables 1 and 2 respectively.

Table 1: The comparison result of theoretical and experimental critical load about stainless steel pipe with sinusoidal buckling.

Specimens		1	2	3	4	5
Experimental value (*N*)		150	160	155	150	150
Theoretical value (*N*)= 169.5	Deviation (%)	-11.5	-5.6	-8.5	-11.5	-11.5
Specimens		6	7	8	9	10
Experimental value (N)		160	150	160	170	150
Theoretical value (N)= 169.5	Deviation (%)	-5.6	-11.5	-5.6	0.3	-11.5

Table 2: The comparison result of theoretical and experimental critical load about stainless steel pipe with helical buckling.

Specimens		1	2	3	4	5
Experimental value (*N*)		1800	1900	1850	2000	1950
Theoretical value (N)= 1945	Deviation (%)	8.1	2.4	5.1	2.8	0.3
Specimens		6	7	8	9	10
Experimental value (*N*)		1925	1950	2000	1800	1950
Theoretical value (N)= 1945	Deviation (%)	1.0	0.3	2.8	8.1	0.3

From Table 1, the energy method is used to calculate the critical load for stainless steel pipe in this article. When considering the power applied by gravity, the critical load of sinusoidal buckling is 169.5 N. Without considering it, the value is 169.2 N. The difference between two results is very small. Therefore, the influence of gravity can be ignored in the calculation of critical buckling load. As can be seen in the above table, the error in the theoretical and experimental critical load is less than 15% in 10 specimens with sinusoidal buckling. From Table 2, the error in the theoretical and experimental critical load is less than 10% in 10 specimens with helical buckling.

The Comparison Results of Friction Resistance with Experimental and Theoretical Calculation for Slender Columns with Buckling

In this paper, the frictional resistances are calculated for 1-10 group specimens with sinusoidal and helical buckling. The values are compared with the experimental values. The results are shown in Tables 3 and 4 respectively. The contrast curve is shown in Figure 5.

From Table 3, Table 4, and Figure 5 (a), the measurement point for sinusoidal buckling is 92 in 10 group specimens of stainless steel. For 90 points, the error is less than 15%. The error in only two measuring points is greater than 20%. It is obvious from Table 3, Table 4, and Figure 5 (b) that the measurement points for helical buckling is 36 in 10 group specimens of stainless steel. The error in all measuring points is less than 10%. The experimental and theoretical values coincide with each other. To sum up, Equations 21 and 23 can describe the friction resistance value for the slender rod string with sinusoidal and helical buckling well.

In order to compare the impact of gravitational potential energy and friction resistance dissipated energy, the stainless steels are also chosen.

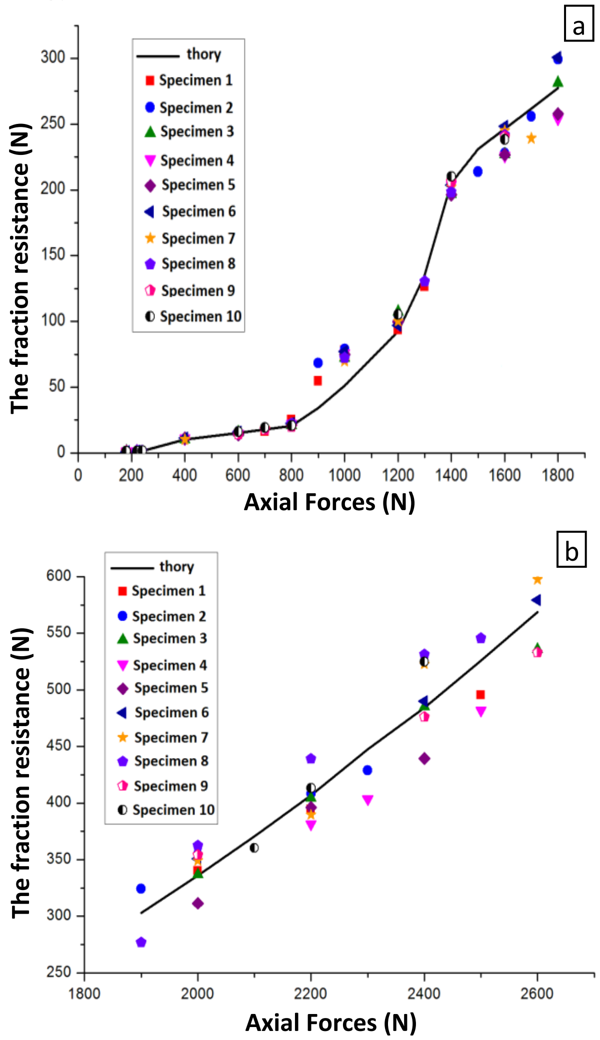


Figure 5: The comparison curve of theoretical and experimental frictional force about stainless steel pipe with helical buckling; (a) sine, (b) helix.

An axial pressure of 1.5 kN was applied to the bottom of the string. The coefficient is 0.5 between the stainless steel pipe and the organic glass tube. The stainless steel tubes with the length of 10 m, 20 m, 30 m, 40 m, 50 m, 70 m, and 100 m are calculated respectively. The calculation results are shown in Table 5.

As it can be seen from Table 5, when the gravitational potential energy and friction resistance dissipated energy are considered, the friction resistance value gradually increases with the length of pipe, and its value increased from 1.82 kN to 32.03 kN. While the gravitational potential energy and friction resistance dissipated energy are neglected, the trend of friction resistance value is the same, but its value increased from 1.75 kN to 18.53 kN. Thus, the friction resistance values increase rapidly when one considers gravitational potential energy and friction resistance dissipated energy. Therefore, the gravitational potential energy and friction resistance dissipated energy must be taken into account in the calculation of friction resistance for slender rod string in wellbore.

CONCLUSIONS

According to the results obtained the following conclusions can be drawn:

1. The slender rod string is selected as a research object in vertical wells. The critical load of sinusoidal buckling and helical buckling with bottom hole pressure was derived based on the energy method.

2. According to the geometry of sinusoidal and helical buckling, the gravitational potential energy and friction damping dissipation were introduced in Lagrange multiplier method. The calculation method of contact force and friction resistance was established for slender rod string in the article.

3. Aiming the problem of sinusoidal and helical buckling, the specialized experimental device was developed for column buckling. The change rules of sinusoidal and helical buckling were obtained for columns under different loads, and the theoretical results coincided with the theoretical ones.

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REFERENCES

[1] Tan M. L., Gan L. F., and Wang X. W., “Nonlinear Helical Buckling of Drill- Strings in Inclined Wellbore,” *Engineering Mechanics*, **2007**, *24*, 199-202.

[2] Mitchell R. F., “Buckling of tubing inside casing,” SPE 150613, IADC/SPE Drilling Conference and Exhibition, San Diego, California, USA, **2012**, 1-13.

[3] Xiu-gen W., Bai-lin Z., and Peng-fei H., “Study on Post-buckling of Constrained Bar,” *Journal of Tongji University* (Natural Science), **2009**, *37*, 26-31.

[4] Gao G. and Miska S., “Effects of Friction on Post-buckling Behavior and Axial Load Transfer in a Horizontal Well,” *SPF Journal*, SPE 120084, **2010**, 15, 1104-1118.

[5] Gao G. and Di Q., Miska S., “Stability Analysis of Pipe with Connectors in Horizontal Wells,” *SPF Journal* SPE 146959, **2012**, *17*, 931-941.

[6] Jubao L., Xiuwen W., and Qianbei Y., “The Buckling Deformation and Contact with Inner Rod and Outer Casing for Compressional Tubing,” *Mechanics in Engineering*, **2011**, *33*, 25-29.

[7] Tan M. and Gan L., “Equilibrium Equations for Nonlinear Buckling Analysis of Drill-strings in 3D Curved Wellbores,” *Science in China Series E: Technological Sciences*, **2009**, *52*, 590-595.

[8] Mitchell R. F., “Tubing Buckling: the State of The Art,” *SPE Drilling & Completion,* SPE 104267, **2008**, *23*, 361 - 370.

[9] Mitchell R. F. and Weltzin T., “Lateral Buckling: the Key to Lockup,” SPE 139824, *SPE Drilling & Completion,* **2011**, *26*, 436-450.