

Non-Darcian Mixed Convection Flow in Vertical Composite Channels with Hybrid Boundary Conditions

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Abstract

In this article, the effects of viscous dissipation and inertial force on the velocity and temperature distributions of the mixed convection laminar flow in a vertical channel partly filled with a saturated porous medium have been studied. In this regard, the Brinkman–Forchheimer extended Darcy model was adopted for the fluid flow in the porous region. In addition, three different viscous dissipation models with isoflux-isothermal boundary conditions were applied. To determine the velocity and temperature distributions for both the regions, the coupled non-linear governing equations were solved using two parameter perturbation and numerical methods. Moreover, the results of the numerical method were validated against those predicted by the perturbation method for small values of the dimensionless perturbation parameters. Furthermore, the results obtained for both regions were compared in terms of Grashof, Reynolds, Forchheimer, and Brinkman numbers. The predicted results clearly indicate that the type of viscous dissipation model has a significant effect on the temperature and velocity distributions.

Key words: Composite Channel; Perturbation Method; Isoflux Boundary; Inertial Effect; Viscous Dissipation.

INTRODUCTION

Convective heat transfer in closed conduits partially filled with a porous medium is of essential importance to a variety of engineering applications including solar collectors, micro scale channels for cooling electronic components, nuclear reactors, chemical catalytic reactors, thermal insulation, and heat pipes. In the past decade, this importance has attracted substantial analytical studies. Fluid flow and convective heat transfer in a system simultaneously containing a fluid reservoir and a porous medium saturated with a fluid is of great mathematical and physical interest. More specifically, the existence of a fluid layer adjacent to a layer of fluid saturated porous medium is a common occurrence in both geophysical and engineering environment [1].

Viscous dissipation can cause an appreciable rise in the fluid temperature due to the conversion of kinetic energy of the fluid to the thermal energy and is considered as a source term in the fluid flow. This effect is of particular significance in high speed fluid flows, flows of highly viscous fluids [2], flows through conventionally sized

channels with large length to width ratio, and flows in microchannels [3]. Although the same situation prevails in porous media [4], most existing analytical studies of forced and mixed convection in porous media neglect the effect of viscous dissipation [5–12].

For fully developed forced convection in porous media, Nield et al [13] investigated the effects of viscous dissipation in a parallel plate channel filled with a porous medium for both isoflux and isothermal walls. The values of Nusselt number for different cases were extensively presented as a function of Darcy and Brinkman numbers. Ranjbar-Kani and Hooman [14] and Hooman and Gurgenci [15] numerically studied the forced convection with viscous dissipation in a saturated porous medium. Analytical expressions for the asymptotic temperature profile and the asymptotic Nusselt number values for the isothermal boundary conditions were also presented for verifying the numerical results in which the convective term was neglected in the thermal energy equation for fully developed fluid flow.

As mentioned earlier, there are numerous analytical

studies reported in the literature on the forced convection in porous media, but few studies deal with the effect of viscous dissipation on the transverse temperature distribution within a porous medium. However, the temperature variation within a porous medium is of primary importance for understanding the mechanism of heat transfer processes and predicting the heat-transfer rate [16].

In the literature on fluid flow in media, which are totally filled with porous materials, it has been reported that the local macroscopic inertial term is usually small compared to the microscopic Darcy drag term and hence it can be neglected. In most practical situations, the velocity responds to an imposed pressure drop within a second or less. The local inertial term may be important if an oscillatory pressure gradient is imposed or if the porous domain is of large void fraction. However, it is obvious that the local inertial term may retain its importance in applications involving very thin porous substrates or at large Darcy number. Therefore, the study of convective heat transfer in such coarse porous media requires generalized momentum equations [17,18]. For example, Abu-Hijleh and Al-Nimr [18] investigated the importance of the local inertial term in the forced convection and fluid flow problems in channels partially filled with a porous medium. Vafai and Tien [19] studied boundary and inertial effects of porous media, which are not normally taken into account in the well known Darcy's law. Umavathi et al [17-20] conducted a numerical study on the combined convection in a vertical channel filled with a porous medium by considering the effect of inertial force.

The major objective of the present study was to study the details of transverse temperature variations in a parallel plate channel partly filled with a saturated porous medium. In this regard, the governing equations subject to isoflux/isothermal boundary conditions were solved using approximate analytical and numerical methods to obtain the distributions of velocity and temperature in terms of Grashof, Forchheimer and Brinkman numbers.

STATEMENT OF THE MODEL

Let us consider a Newtonian fluid, which steadily flows in a parallel plate vertical channel partly filled with a saturated porous medium in which combined forced and natural convection takes place. As shown in Figure 1, the x axis was chosen as the direction of fluid flow against the gravitational field whereas the y axis was transverse to the walls. Moreover, suppose that both the fluids flowing in the porous and viscous regions are incompressible and the porous medium is isotropic and homogeneous.

At the walls of the channel, a thermal boundary condition either of the first kind (i.e., temperature distribution) or of the second kind (i.e., constant wall heat flux) could be prescribed. Obviously, at both walls of the channel, all the combinations of the above conditions and even their self combinations can be allowed [21].

The isoflux and the constant wall temperature conditions at the left and right walls of the channel correspond to the hybrid combination of the second and first kind of thermal boundary conditions, respectively. Therefore, the right wall of the channel was kept at temperature T_w , whereas the left wall was exposed to a constant wall heat flux q_w .

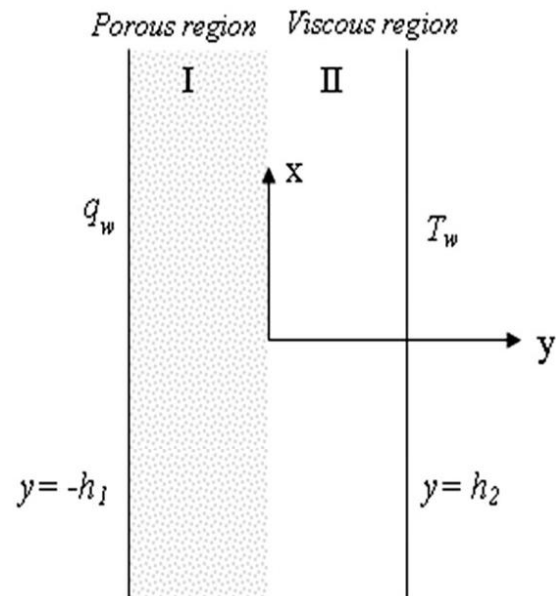


Figure 1.: Configuration of the problem and its boundary conditions

The thermo physical properties of the fluid and effective properties of the porous medium were assumed to be constant, except for the fluid density applied in the buoyancy term in the momentum equations [22]. Moreover, the Oberbeck-Boussinesq approximation was used throughout the present work and the fluid within the porous medium and the solid matrix were in the local thermal equilibrium.

According to Figure 1, the region $-h_1 \leq y \leq 0$ was filled with a saturated porous medium with the density ρ_1 , viscosity μ_1 , thermal conductivity k_1 , and the thermal expansion coefficient β_1 , whereas the region $0 \leq y \leq h_2$ is a viscous fluid with the density ρ_2 , viscosity μ_2 , thermal conductivity k_2 , and the thermal expansion coefficient β_2 .

Governing Equations

In this section, the momentum and energy balance equations are written in dimensionless forms. Then, the solutions to these equations for three different cases are presented. Consider a steady state, laminar, and fully developed fluid flow in the parallel plate channel in which the only nonzero component of the velocity field is its longitudinal component, u , (i.e., x component of velocity field). Under these conditions, the continuity equations for both the regions become:

$$\frac{\partial u_i}{\partial x} = 0 \quad i = 1, 2 \quad (1)$$

Moreover, equations of motion for both the regions can

be expressed as [23]:

1- Porous region:

x direction:

$$g\beta_1(T_1 - T_0) - \frac{1}{\rho_1} \frac{\partial P_1}{\partial x} + v_{eff} \frac{d^2 u_1}{dy^2} - \frac{v_1}{K} u_1 - \frac{C_F}{\sqrt{K}} u_1^2 = 0 \quad (2)$$

$$y \text{ direction: } \frac{\partial P_1}{\partial y} = 0 \quad (3)$$

2- Viscous region:

$$x \text{ direction: } g\beta_2(T_2 - T_0) - \frac{1}{\rho_2} \frac{\partial P_2}{\partial x} + v_2 \frac{d^2 u_2}{dy^2} = 0 \quad (4)$$

$$y \text{ direction: } \frac{\partial P_2}{\partial y} = 0 \quad (5)$$

where $g\beta(T - T_0)$, $v_{eff}(d^2 u/dy^2)$, and $(v/K) u$ are the buoyancy term, viscous term, and the Darcy term, respectively, whereas accounts for the inertial force and finally $P = p + \rho g x$ is the hydrodynamic pressure. Considering equations 3 and 5, equations 2 and 4 can be rewritten as:

$$g\beta_1(T_1 - T_0) - \frac{1}{\rho_1} \frac{dP_1}{dx} + v_{eff} \frac{d^2 u_1}{dy^2} - \frac{v_1}{K} u_1 - \frac{C_F}{\sqrt{K}} u_1^2 = 0 \quad (6)$$

$$g\beta_2(T_2 - T_0) - \frac{1}{\rho_2} \frac{dP_2}{dx} + v_2 \frac{d^2 u_2}{dy^2} = 0 \quad (7)$$

where T_0 is a reference temperature. Moreover, if one assumes that there is a constant A such that:

$$\frac{dP_i}{dx} = A; i = 1 \text{ and } 2 \quad (8)$$

then taking the derivatives of equations 6 and 7 with respect to x and using eq 8 gives:

$$\frac{dT_i}{dx} = 0; i = 1 \text{ and } 2 \quad (9)$$

Equation 9 clearly shows that the temperature of fluids is a function of y alone as well.

Considering equation 9, the thermal energy equation for the region I taking into account the effect of viscous dissipation is given by:

$$k_1 \frac{d^2 T_1}{dy^2} + \Phi = 0 \quad (10)$$

$$\Phi = \frac{\mu_1}{K} u_1^2 - c_1 \mu_{eff} u_1 \frac{d^2 u_1}{dy^2} + c_2 \mu_1 \left(\frac{du_1}{dy} \right)^2$$

where Φ is the viscous heating due to viscous dissipation in the porous medium. In the literature, three models proposed for viscous dissipation in porous media are as follows: [13]

(1) For model 1 (i.e., Darcy model), $c_1=0$ and $c_2=0$

(2) For model 2 (i.e., power of drag force model), $c_1=1$ and $c_2=0$

(3) For model 3 (i.e., clear fluid compatible model), $c_1=0$ and $c_2=1$.

The conservation equations for the porous region are based on the non-Darcian model taking into account the Forchheimer-Brinkman extension in the momentum equation as well as the viscous dissipation terms in the thermal energy equations. Note that for high permeability

porous media, the Brinkman extension must be included [24]. However, for moderate values of the velocity and viscosity of the fluids, the dissipation terms are important and therefore have to be included in the thermal energy equations.

When the flow in an isotropic porous medium satisfies Darcy's law, the appropriate heat source term that models viscous dissipation in the thermal energy equation is given by the Darcy model (i.e., Model 1). Nield [25] stated that this form of Φ can be obtained by substituting $\Phi = u \cdot F$, where F is the drag force on the porous medium. Thus, if Darcy's law is valid and the permeability is isotropic, then $F = (\mu/K) u$. While the form of Φ given by Model (1) is widely accepted for the Darcy flow, the same cannot be said for flows where boundary effects, as modeled by the Brinkman terms, are significant. Nield's drag force formula yields the form given by Model (2) while Al Hadhrami et al [26]. used an argument based on the work done by frictional forces to obtain Model (3) in one dimensional flow. Both formulae yield the correct form of Φ in the case of small permeability porous media. However, when the porosity increases to 1, only the formula of Al-Hadhrami et al [26]. matches that for a clear fluid [27].

$$k_2 \frac{d^2 T_2}{dy^2} + \mu_2 \left(\frac{du_2}{dy} \right)^2 = 0 \quad (11)$$

where the second term accounts for the effect of viscous heating.

Note that the momentum equations for the porous layer (i.e., equations 2 and 3) are based on the non-Darcian model considering the Brinkman and Forchheimer terms. Beckermann et al [24] have experimentally shown that in the natural convection in vertical channels containing fluid and porous layers, the Brinkman term is small compared to the Darcy term. However, they considered the Brinkman term in all their simulations to ensure continuity of velocities and stresses at the interface of the fluid/porous medium. Both the Brinkman and Forchheimer terms should be taken into account for high permeability porous media, i.e., high Darcy number [17].

Equations 6 and 7 can be transformed into the following equations by using eqs 10 and 11:

$$\frac{d^4 u_1}{dy^4} = \left(\frac{c_2 \rho_1 g \beta_1 \mu_1}{\mu_{eff} k_1} + \frac{2C_F \rho_1}{\mu_{eff} \sqrt{K}} \right) \left(\frac{du_1}{dy} \right)^2 + \left(\frac{\rho_1 g \beta_1 \mu_1}{\mu_{eff} k_1 K} \right) u_1^2 - \left(\frac{c_1 \rho_1 g \beta_1}{k_1} - \frac{2C_F \rho_1}{\mu_{eff} \sqrt{K}} \right) u_1 \frac{d^2 u_1}{dy^2} + \left(\frac{\mu_1}{\mu_{eff} K} \right) \frac{d^2 u_1}{dy^2} \quad (12)$$

$$\frac{d^4 u_2}{dy^4} = \left(\frac{\rho_2 g \beta_2}{k_2} \right) \left(\frac{du_2}{dy} \right)^2 \quad (13)$$

Boundary Conditions

The two regions are coupled by equating the velocity and the shear stress for the momentum equations while the matching of the temperature and heat flux is taken for the thermal energy equations. The governing equations

have to be solved subject to the following boundary conditions:

$$\text{at } y = -h_1 : u_1 = 0, \quad -k_1 \frac{\partial T_1}{\partial y} = q_w$$

$$\text{at } y = h_2 : u_2 = 0, \quad T_2 = T_w$$

$$\text{at } y = 0 : u_1 = u_2, \quad T_1 = T_2$$

$$\text{at } y = 0 : \mu_1 \frac{du_1}{dy} = \mu_2 \frac{du_2}{dy}, \quad -k_1 \frac{\partial T_1}{\partial y} = -k_2 \frac{\partial T_2}{\partial y} \quad (14)$$

To transform these equations into dimensionless forms, we introduced the following dimensionless parameters:

$$\begin{aligned} U_1 &= \frac{u_1 \mu_1}{A h_1^2}, \quad U_2 = \frac{u_2 \mu_2}{A h_2^2}, \quad Y_1 = \frac{y}{h_1}, \\ Y_2 &= \frac{y}{h_2}, \quad \theta_1 = \frac{(T_1 - T_0)}{\Delta T}, \quad \theta_2 = \frac{(T_2 - T_0)}{\Delta T} \\ Gr &= \frac{g \beta_1 \Delta T h_1^3}{\nu_1^2}, \quad Re = \frac{A h_1^3 \rho_1}{\mu_1^2}, \quad Br = \frac{A^2 h_1^4}{\mu_1 k_1 \Delta T}, \\ Da &= \frac{K}{h_1^2}, \quad M = \frac{\mu_{eff}}{\mu_1}, \quad \sigma = \frac{1}{\sqrt{Da M}} \end{aligned} \quad (15)$$

Using these dimensionless parameters, equations 12 and 13 become:

$$\begin{aligned} \frac{d^4 U_1}{dY^4} - \sigma^2 \frac{d^2 U_1}{dY^2} &= \frac{2F}{M} \left[\left(\frac{dU_1}{dY} \right)^2 + U_1 \frac{d^2 U_1}{dY^2} \right] + \\ GR Br \left[\sigma^2 U_1^2 + \frac{c_2}{M} \left(\frac{dU_1}{dY} \right)^2 - c_1 U_1 \frac{d^2 U_1}{dY^2} \right] \end{aligned} \quad (16)$$

$$\frac{d^4 U_2}{dY^4} = m n \kappa b h^4 GR Br \left(\frac{dU_2}{dY} \right)^2 \quad (17)$$

subject to the following dimensionless boundary conditions:

$$\text{at } Y = -1 : U_1 = 0, \quad \frac{d^3 U_1}{dY^3} - \sigma^2 \frac{dU_1}{dY} - \frac{2F}{M} U_1 \frac{dU_1}{dY} = \frac{GR}{M}$$

$$\text{at } Y = 1 : U_2 = 0, \quad \frac{d^2 U_2}{dY^2} = 1 - n b R_q GR$$

$$\text{at } Y = 0 : U_1 = m h^2 U_2, \quad \frac{d^2 U_1}{dY^2} - \sigma^2 U_1 - \frac{F}{M} U_1^2 = \left(\frac{1}{M n b} \right) \left[\frac{d^2 U_2}{dY^2} + n b - 1 \right]$$

$$\text{at } Y = 0 : \frac{dU_1}{dY} = h \frac{dU_2}{dY}, \quad \frac{d^3 U_1}{dY^3} - \sigma^2 \frac{dU_1}{dY} - \frac{2F}{M} U_1 \frac{dU_1}{dY} = \frac{1}{n b \kappa h M} \frac{d^3 U_2}{dY^3} \quad (18)$$

SOLUTION

In what follows, we present analytical approximate solutions for the governing equations.

Case I. Negligible Forchheimer and Brinkman Terms (i.e., $Br = 0$ and $F = 0$)

In the case where both the inertial force and viscous heating are negligible, the velocity distribution of the fluid in both the regions can be analytically determined by integrating eqs 16 and 17 subject to the dimensionless boundary conditions given by eq 18 with both F and Br

set equal to 0. The velocity distributions for this case become:

$$U_1 = a_1 + a_2 Y + a_3 \cosh(\sigma Y) + a_4 \sinh(\sigma Y) \quad (19)$$

$$U_2 = b_1 + b_2 Y + b_3 Y^2 + b_4 Y^3 \quad (20)$$

where parameters a_i and b_i ($i = 1, \dots, 4$) are the constants of integration. Moreover, using the dimensionless parameters given by eq 15, the thermal energy equations for both the regions become:

$$\theta_1 = \frac{M}{GR} \left(\frac{1}{M} - \frac{d^2 U_1}{dY^2} + \sigma^2 U_1 \right) = \frac{M}{GR} \left(\frac{1}{M} + \sigma^2 a_1 + \sigma^2 a_2 Y \right) \quad (21)$$

$$\theta_2 = \frac{1}{n b GR} \left(1 - \frac{d^2 U_2}{dY^2} \right) = \frac{1}{GR n b} (1 - 2 b_3 - 6 b_4 Y) \quad (22)$$

The integration constants for $m=n=b=-h=Rqt=1$, and $GR=10$ are as follows:

$$\begin{aligned} a_1 &= 4.75, \quad a_2 = -2.5, \quad a_3 = 0.52, \quad a_4 = 2.53 \\ b_1 &= 5.26, \quad b_2 = 2.57, \quad b_3 = -9.5, \quad b_4 = 1.67 \end{aligned} \quad (23)$$

Case II. Negligible Brinkman Term only (i.e., $Br = 0$ and $F \neq 0$)

In this case, only the effect of viscous heating is neglected and the effect of inertial force is taken into account. Therefore, equations 16 and 17 become:

$$\frac{d^4 U_1}{dY^4} - \sigma^2 \frac{d^2 U_1}{dY^2} = \frac{2F}{M} \left[\left(\frac{dU_1}{dY} \right)^2 + U_1 \frac{d^2 U_1}{dY^2} \right] \quad (24)$$

$$\frac{d^4 U_2}{dY^4} = 0 \quad (25)$$

Solutions to eqs 24 and 25 subject to the corresponding boundary conditions (i.e., eq 18) can be obtained by a regular perturbation method. The approximate solution to these equations for small values of F can be expressed as:

$$U(Y) = U_0(Y) + \gamma U_1(Y) + \gamma^2 U_2(Y) + \dots = \sum_{n=0}^{\infty} \gamma^n U_n(Y) \quad (26)$$

The dimensionless perturbation parameter can be defined by:

$$\gamma = \frac{2F}{M} \quad (27)$$

Upon substitution of eq 26 for $n=0$ and 1 into eqs 24, 25, and 18, the momentum equations for both the regions become:

Porous region:

$$\frac{d^4 U_{10}}{dY^4} - \sigma^2 \frac{d^2 U_{10}}{dY^2} = 0 \quad (28)$$

$$\frac{d^4 U_{11}}{dY^4} - \sigma^2 \frac{d^2 U_{11}}{dY^2} = \left[\left(\frac{dU_{10}}{dY} \right)^2 + U_{10} \frac{d^2 U_{10}}{dY^2} \right] \quad (29)$$

Viscous region:

$$\frac{d^4 U_{20}}{dY^4} = 0 \quad (30)$$

$$\frac{d^4 U_{21}}{dY^4} = 0 \quad (31)$$

subject to the following boundary conditions:

Zeroth-order equations:

$$\begin{aligned} \text{at } Y = -1: \quad U_{10} &= 0, \quad \frac{d^3 U_{10}}{dY^3} - \sigma^2 \frac{dU_{10}}{dY} = \frac{GR}{M} \\ \text{at } Y = 1: \quad U_{20} &= 0, \quad \frac{d^2 U_{20}}{dY^2} = 1 - nb R_{qt} GR \\ \text{at } Y = 0: \quad U_{10} &= m h^2 U_{20}, \quad \frac{d^2 U_{10}}{dY^2} - \sigma^2 U_{10} = \left(\frac{1}{Mnb} \right) \left[\frac{d^2 U_{20}}{dY^2} + nb - 1 \right] \\ \text{at } Y = 0: \quad \frac{dU_{10}}{dY} &= h \frac{dU_{20}}{dY}, \quad \frac{d^3 U_{10}}{dY^3} - \sigma^2 \frac{dU_{10}}{dY} = \frac{1}{nb\kappa h M} \frac{d^3 U_{20}}{dY^3} \end{aligned} \quad (32)$$

First-order equations:

$$\begin{aligned} \text{at } Y = -1: \quad U_{11} &= 0, \quad \frac{d^3 U_{11}}{dY^3} - \sigma^2 \frac{dU_{11}}{dY} - U_{10} \frac{dU_{10}}{dY} = 0 \\ \text{at } Y = 1: \quad U_{21} &= 0, \quad \frac{d^2 U_{21}}{dY^2} = 0 \\ \text{at } Y = 0: \quad U_{11} &= m h^2 U_{21}, \\ \frac{d^2 U_{11}}{dY^2} - \sigma^2 U_{11} - \frac{1}{2} U_{10}^2 &= \left(\frac{1}{Mnb} \right) \frac{d^2 U_{21}}{dY^2} \\ \text{at } Y = 0: \quad \frac{dU_{11}}{dY} &= h \frac{dU_{21}}{dY}, \\ \frac{d^3 U_{11}}{dY^3} - \sigma^2 \frac{dU_{11}}{dY} - U_{10} \frac{dU_{10}}{dY} &= \frac{1}{nb\kappa h M} \frac{d^3 U_{21}}{dY^3} \end{aligned} \quad (33)$$

Solving the zeroth and first order equations for the porous and viscous regions yields:

$$\begin{aligned} U_{10} &= M_1 + M_2 Y + M_3 \cosh(\sigma Y) + M_4 \sinh(\sigma Y) \\ U_{20} &= N_1 + N_2 Y + N_3 Y^2 + N_4 Y^3 \\ U_{11} &= P_1 + P_2 Y + P_3 Y^2 + P_4 \cosh(\sigma Y) + \\ &+ P_5 Y^2 \cosh(\sigma Y) + P_6 Y^2 \sinh(\sigma Y) + \\ &+ P_8 Y^2 \cosh(\sigma Y) + P_9 Y^2 \sinh(\sigma Y) + \\ &P_{10} \cosh(2\sigma Y) + P_{11} \sinh(2\sigma Y) \\ U_{21} &= Q_1 + Q_2 Y + Q_3 Y^2 + Q_4 Y^3 \end{aligned} \quad (34)$$

where M_i , N_i , P_i , and Q_i are the constants of integration. Moreover, the thermal energy equations for both the regions become:

$$\begin{aligned} \theta_1 &= \frac{M}{GR} \left(\frac{1}{M} - \frac{d^2(U_{10} + \gamma U_{11})}{dY^2} + \right. \\ &\left. \sigma^2(U_{10} + \gamma U_{11}) + \frac{F}{M}(U_{10} + \gamma U_{11})^2 \right) \\ \theta_2 &= \frac{1}{nbGR} \left(1 - \frac{d^2(U_{20} + \gamma U_{21})}{dY^2} \right) \end{aligned} \quad (35)$$

Case III: Non-Negligible Forchheimer and Brinkman Terms (i.e., $Br \neq 0$ and $F \neq 0$)

In this general case, the effects of viscous heating and the inertial force are both taken into account. The approximate solutions to equations 16 and 17 can be obtained by a two parameter perturbation method. The dimensionless perturbation parameters can be defined as:

$$\varepsilon = GR Br, \quad \gamma = \frac{2F}{M} \quad (36)$$

Suppose that the values of these perturbation parameters

are small (i.e., $|\gamma| \ll 1$ and $|\varepsilon| \ll 1$). The approximate solutions to equations 16 and 17 for specified values of F and Br can be expressed as:

$$\begin{aligned} U(Y) &= U_0(Y) + \varepsilon U_1(Y) + \gamma V_1(Y) + \varepsilon^2 U_2(Y) + \gamma^2 V_2(Y) + \dots \\ &= U_0(Y) + \sum_{n=1}^{\infty} [\varepsilon^n U_n(Y) + \gamma^n V_n(Y)] \end{aligned} \quad (37)$$

By substituting eq 39 into eqs 16, 17, and 18 for $n = 0$ and 1, the momentum equations for both the regions become:

Porous region:

$$\frac{d^4 U_{10}}{dY^4} - \sigma^2 \frac{d^2 U_{10}}{dY^2} = 0 \quad (38)$$

$$\frac{d^4 U_{11}}{dY^4} - \sigma^2 \frac{d^2 U_{11}}{dY^2} = \left[\frac{c_2}{M} \left(\frac{dU_{10}}{dY} \right)^2 + \sigma^2 U_{10}^2 - c_1 U_{10} \frac{d^2 U_{10}}{dY^2} \right] \quad (39)$$

$$\frac{d^4 V_{11}}{dY^4} - \sigma^2 \frac{d^2 V_{11}}{dY^2} = \left[\left(\frac{dU_{10}}{dY} \right)^2 + U_{10} \frac{d^2 U_{10}}{dY^2} \right] \quad (40)$$

Viscous region:

$$\frac{d^4 U_{20}}{dY^4} = 0 \quad (41)$$

$$\frac{d^4 U_{21}}{dY^4} = mn\kappa b h^4 \left(\frac{dU_{20}}{dY} \right)^2 \quad (42)$$

$$\frac{d^4 V_{21}}{dY^4} = 0 \quad (43)$$

Moreover, the corresponding boundary and interface conditions are:

Zeroth order equations:

$$\text{at } Y = -1: \quad U_{10} = 0, \quad \frac{d^3 U_{10}}{dY^3} - \sigma^2 \frac{dU_{10}}{dY} = \frac{GR}{M}$$

$$\text{at } Y = 1: \quad U_{20} = 0, \quad \frac{d^2 U_{20}}{dY^2} = 1 - nb R_{qt} GR$$

$$\text{at } Y = 0: \quad U_{10} = m h^2 U_{20},$$

$$\frac{d^2 U_{10}}{dY^2} - \sigma^2 U_{10} = \left(\frac{1}{Mnb} \right) \left[\frac{d^2 U_{20}}{dY^2} + nb - 1 \right]$$

$$\text{at } Y = 0: \quad \frac{dU_{10}}{dY} = h \frac{dU_{20}}{dY},$$

$$\frac{d^3 U_{10}}{dY^3} - \sigma^2 \frac{dU_{10}}{dY} = \frac{1}{nb\kappa h M} \frac{d^3 U_{20}}{dY^3} \quad (44)$$

First order equations:

$$\text{at } Y = -1: \quad U_{11} = 0, \quad V_{11} = 0$$

$$\text{at } Y = 1: \quad U_{21} = 0, \quad V_{21} = 0$$

$$\text{at } Y = 0: \quad U_{11} = m h^2 U_{21}, \quad V_{11} = m h^2 V_{21}$$

$$\text{at } Y = -1: \quad \frac{d^3 U_{11}}{dY^3} - \sigma^2 \frac{dU_{11}}{dY} = 0,$$

$$\frac{d^3 V_{11}}{dY^3} - \sigma^2 \frac{dV_{11}}{dY} - U_{10} \frac{dU_{10}}{dY} = 0$$

$$\text{at } Y = 1: \quad \frac{d^2 U_{21}}{dY^2} = 0, \quad \frac{d^2 V_{21}}{dY^2} = 0$$

$$\text{at } Y = 0: \quad \frac{dU_{11}}{dY} = h \frac{dU_{21}}{dY}, \quad \frac{dV_{11}}{dY} = h \frac{dV_{21}}{dY}$$

$$\begin{aligned}
\text{at } Y=0: \quad & \frac{d^2 U_{11}}{dY^2} - \sigma^2 U_{11} = \left(\frac{1}{Mnb} \right) \frac{d^2 U_{21}}{dY^2}, \\
& \frac{d^2 V_{11}}{dY^2} - \sigma^2 V_{11} - \frac{1}{2} U_{10}^2 = \left(\frac{1}{Mnb} \right) \frac{d^2 V_{21}}{dY^2} \\
\text{at } Y=0: \quad & \frac{d^3 U_{11}}{dY^3} - \sigma^2 \frac{dU_{11}}{dY} = \frac{1}{nb\kappa hM} \frac{d^3 U_{21}}{dY^3}, \\
& \frac{d^3 V_{11}}{dY^3} - \sigma^2 \frac{dV_{11}}{dY} - U_{10} \frac{dU_{10}}{dY} = \frac{1}{nb\kappa hM} \frac{d^3 V_{21}}{dY^3} \quad (47)
\end{aligned}$$

The solutions to the zeroth and first order equations for the porous and viscous regions are:

$$\begin{aligned}
U_{10} &= C_1 + C_2 Y + C_3 \cosh(\sigma Y) + C_4 \sinh(\sigma Y) \\
U_{20} &= D_1 + D_2 Y + D_3 Y^2 + D_4 Y^3 \quad (48)
\end{aligned}$$

$$\begin{aligned}
U_{11} &= E_1 + E_2 Y + (E_3 + E_4 c_1 + E_5 c_2) Y^2 + \\
& E_6 Y^3 + E_7 Y^4 + (E_8 + E_9 c_1 + E_{10} c_2) \cosh(\sigma Y) \\
& + (E_{11} + E_{12} c_1 + E_{13} c_2) \sinh(\sigma Y) + \\
& (E_{14} + E_{15} c_1 + E_{16} c_2) Y \cosh(\sigma Y) \\
& + (E_{17} + E_{18} c_1 + E_{19} c_2) Y \sinh(\sigma Y) + (E_{20} + E_{21} c_1) Y^2 \cosh(\sigma Y) \\
& + (E_{22} + E_{23} c_1) Y^2 \sinh(\sigma Y) + (E_{24} + E_{25} c_1 + E_{26} c_2) \sinh(\sigma^2 Y) \\
& + (E_{27} + E_{28} c_1 + E_{29} c_2) \cosh(\sigma^2 Y) + E_{30} e^{\sigma Y} + E_{31} e^{-\sigma Y} \\
U_{21} &= F_1 + F_2 Y + F_3 Y^2 + F_4 Y^3 + \\
& F_5 Y^4 + F_6 Y^5 + F_7 Y^6 + F_8 Y^7 + F_9 Y^8 \quad (49)
\end{aligned}$$

$$\begin{aligned}
V_{11} &= G_1 + G_2 Y + G_3 Y^2 + G_4 \cosh(\sigma Y) + \\
& G_5 \sinh(\sigma Y) + G_6 Y \cosh(\sigma Y) \\
& + G_7 Y \sinh(\sigma Y) + G_8 Y^2 \cosh(\sigma Y) + \\
& G_9 Y^2 \sinh(\sigma Y) + G_{10} \cosh(2\sigma Y) \\
& + G_{11} \sinh(2\sigma Y) \\
V_{21} &= H_1 + H_2 Y + H_3 Y^2 + H_4 Y^3 \quad (50)
\end{aligned}$$

where C_i , D_i , E_i , F_i , G_i , and H_i are the constants of integration.

Furthermore, using the dimensionless parameters given by eq 15, the thermal energy equations for the porous and viscous regions are:

Porous region:

$$\begin{aligned}
\theta_1 &= \frac{M}{GR} \left(\frac{1}{M} - \frac{d^2 (U_{10} + \varepsilon U_{11} + \gamma V_{11})}{dY^2} + \right. \\
& \left. \sigma^2 (U_{10} + \varepsilon U_{11} + \gamma V_{11}) + \frac{F}{M} (U_{10} + \varepsilon U_{11} + \gamma V_{11})^2 \right) \quad (51)
\end{aligned}$$

Viscous region:

$$\theta_2 = \frac{1}{nbGR} \left(1 - \frac{d^2 (U_{20} + \varepsilon U_{21} + \gamma V_{21})}{dY^2} \right) \quad (52)$$

Finally, by obtaining the temperature profiles for both the regions, convective heat transfer coefficient can be determined. The Nusselt number values for the left and

right walls can be evaluated by:

$$Nu_1 = \frac{q_w (h_1 + h_2)}{k_1 (T_1 - T_0)} = \frac{(1 + h)}{\theta_1 (-1)} \quad (53)$$

$$Nu_2 = \frac{(dT_2/dy)(h_1 + h_2)}{\Delta T} = \left(1 + \frac{1}{h} \right) \left(\frac{d\theta_2}{dY} \right)_{Y=1} \quad (54)$$

In addition to the approximate analytical solution, the velocity and temperature distributions in the presence of inertial force and viscous dissipation were obtained using a numerical procedure and the predicted results were compared with the results predicted by the present perturbation method. In this regard, coupled governing equations subject to the appropriate boundary conditions were solved using finite difference method. The derivatives in the equations were replaced with corresponding central difference schemes. The following convergence criteria for solving the obtained algebraic equations were used:

$$\frac{\max |u_{i,j}^{n+1} - u_{i,j}^n|}{\max |u_{i,j}^{n+1}|} \leq 10^{-6}, \quad \frac{\max |T_{i,j}^{n+1} - T_{i,j}^n|}{\max |T_{i,j}^{n+1}|} \leq 10^{-6} \quad (55)$$

where n is the n th iteration.

RESULTS AND DISCUSSION

In the present work, the perturbation solution was compared against the exact solution of the governing equations in the absence of inertial effects in the equations of motion and the viscous dissipation term in the thermal energy equation as shown in Figure 2. It can be observed that the perturbation and the exact solutions are in good agreement.

Figure 3a shows the variations of dimensionless velocity for both the regions with the perturbation parameter, ε , and using different types of viscous dissipation models (i.e., Models 1, 2, and 3) for $\gamma = 0.01$, $GR=10$, and $m=n=b=\kappa=h=R_{qt}=1$. As observed from this figure, the dimensionless velocities for both the regions increase with increasing parameter ε . This behavior can be explained by greater thermal energy generated by the viscous dissipation, which enhances the fluid temperature and consequently results in greater buoyancy force. Therefore, an increase in the buoyancy force increases the velocity in the upward direction.

According to the definition of parameter ε (i.e., $GR \times Br$), at a constant value of GR (i.e., 10), increasing ε increases the Brinkman number. The Brinkman number represents the effect of viscous dissipation such that large values of Br show that more heat dissipates in the medium. Heat dissipation can act as a heat source in the medium and increases the fluid temperature.

In addition, it is interesting to note that no flow reversal occurs at the channel walls compared with constant wall temperature conditions in which, there is flow reversal at the cold wall for the same parameters (i.e., $GR = 10$ and $\gamma=0.01$) [28].

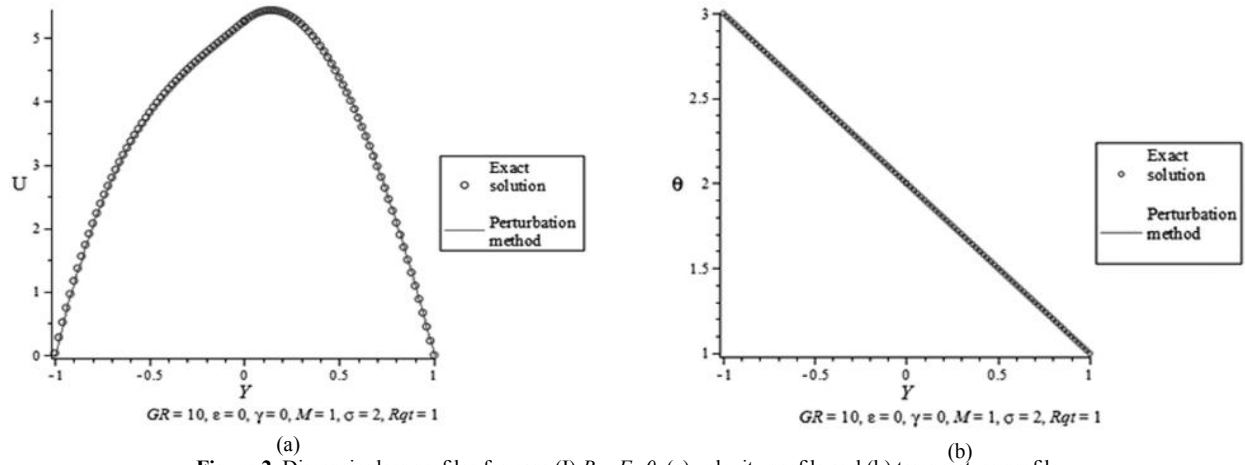


Figure 2. Dimensionless profiles for case (I) $Br=F=0$. (a) velocity profile and (b) temperature profile

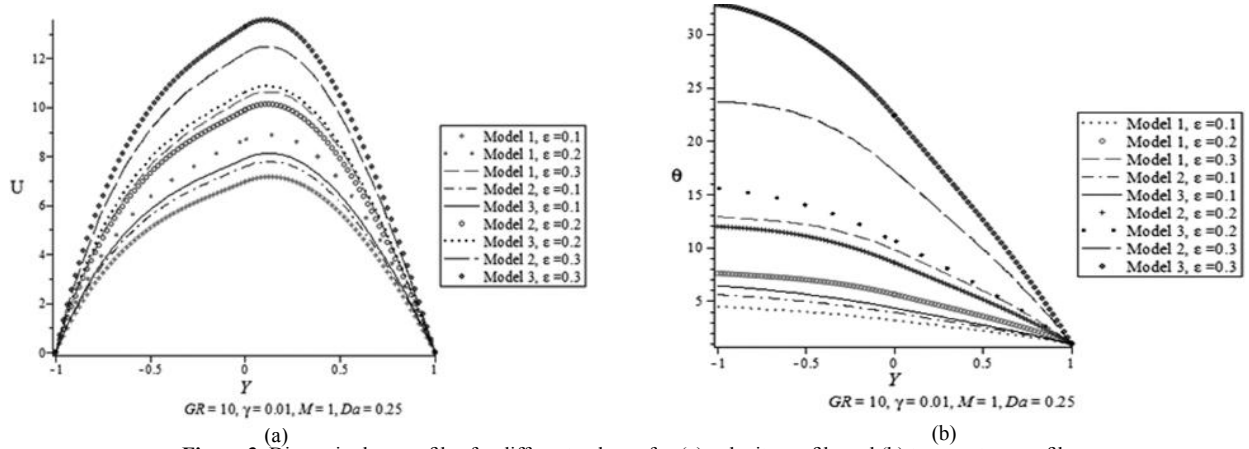


Figure 3. Dimensionless profiles for different values of ε . (a) velocity profile and (b) temperature profile

In fact, viscous dissipation (with increasing ε) increases the buoyancy force and therefore it tends to increase the flow field in the channel. Furthermore, if the inertial force is neglected, the predicted results become similar to those reported by Kumar et al [29]. Besides, this figure shows that the velocity distribution of the fluid varies with various types of viscous dissipation models. It can be seen that the difference between the velocity profiles using different viscous dissipation models increases as ε increases.

Figure 3b demonstrates the variations of dimensionless temperature of both the regions with the parameter ε for $\gamma = 0.01$, $GR = 10$, and $Da = 0.25$ and for different types of viscous dissipation models. This figure clearly shows that the dimensionless temperatures increase with increasing ε . However, the variation of the dimensionless temperature due to the different viscous dissipation models is not significant for small values of ε .

Figures 4a and b demonstrate the variations of dimensionless velocity and temperature for both the porous and viscous regions with the perturbation parameter γ , for $\varepsilon = 0.1$, $Da = 0.25$, and $GR = 10$, respectively. These figures clearly show that dimensionless velocity (U) and the dimensionless temperature (θ) decrease with increasing γ (i.e., F -to- M ratio). The presence of porous medium increases flow resistance. In addition, the inertial effects (with increasing γ) enhance this resistance, which further reduces the flow velocity in the channel.

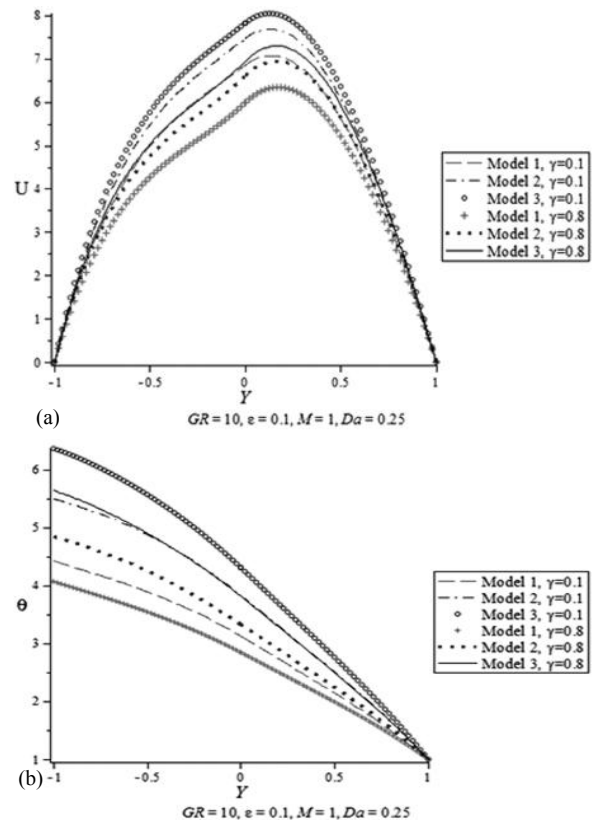


Figure 4. Dimensionless profiles for different values of γ . (a) Velocity profile and (b) temperature profile

Figures 5a and b show the velocity and temperature distributions for both the regions as a function of mixed convection parameter (i.e., GR) for $Br=0.02$ and $\gamma=0.01$. As it may be noticed, GR is a parameter for comparing the intensities of natural and forced convection effects.

In fact, for $GR > 1$, flow is dominated by the natural convection whereas for $GR < 1$, forced convection can be dominant. Thus, at $GR = 1$, the effects of natural and forced convection achieve equal importance and flow is under mixed convection conditions. The trends of the dimensionless velocity and temperature profiles with increasing GR can be explained by an increase in the buoyancy force, which enhances the flow velocity and fluid temperature. The effect of Brinkman number on the Nusselt number values of the channel walls is shown in Figure 6 for different viscous dissipation models.

observed from this figure, the effect of Br on the Nusselt number values is similar to that on the temperature distribution.

The Nusselt number values at $Y = 1$ (i.e., Nu_2) increase with increasing the Brinkman number. On the other hand, Nu_1 is a decreasing function of the Br . The positive sign of Nusselt number values can be explained by the direction of the heat flux at the left and right walls of the channel. As Brinkman number increases, viscous dissipation in the fluid channel increases and therefore the fluid temperature increases. Thus, the temperature difference between the fluid and the left wall of the channel decreases and Nu_1 reduces. Moreover, by increasing the fluid temperature as the result of viscous heating, the fluid and the right wall temperature gradient increases and Nu_2 increases, consequently.

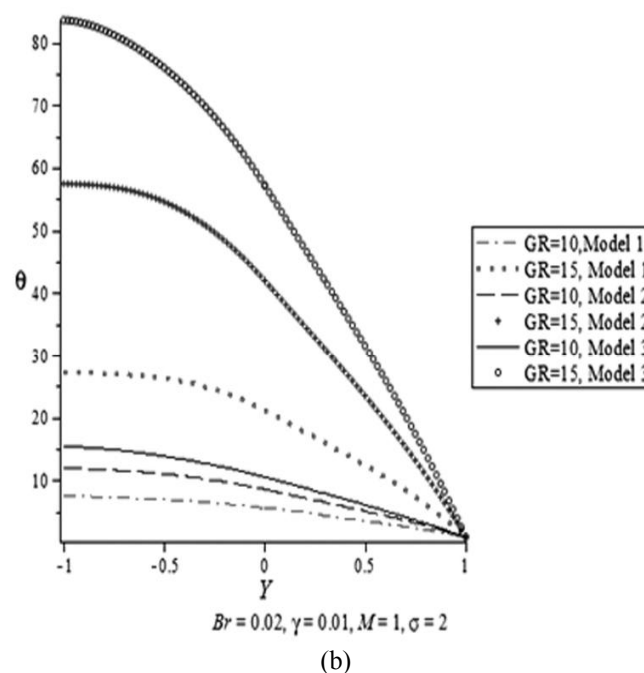
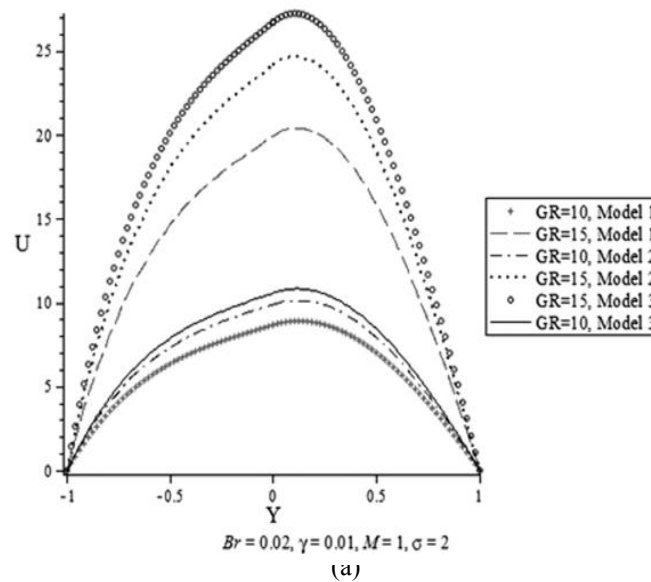


Figure 5. Effects of mixed convection parameter on (a) velocity profile and (b) temperature profile

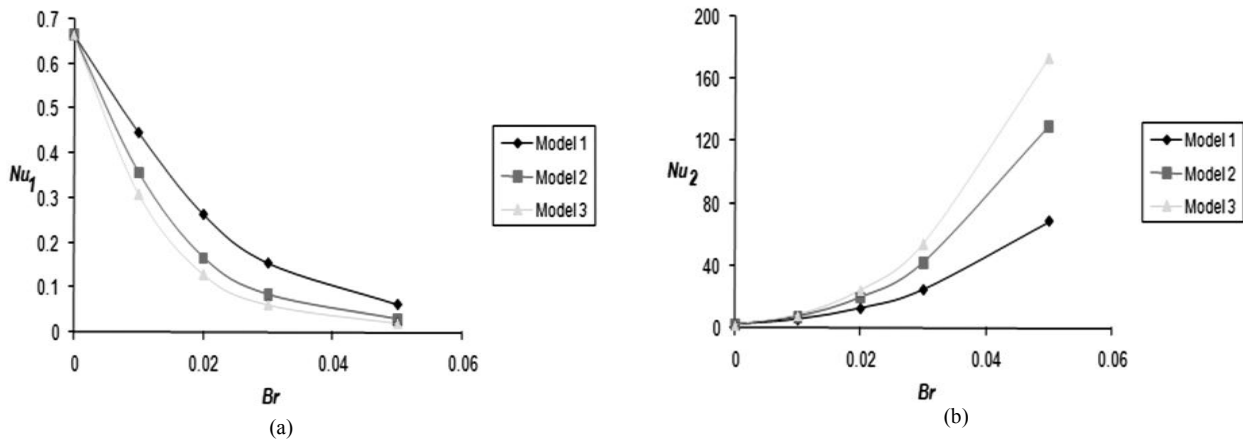


Figure 6. Effects of Brinkman number on (a) Nu_1 and (b) Nu_2

The effect of Forchheimer number in the form of different values of parameter F on the Nusselt number values of the channel walls is shown in Figure 7. This figure shows that the Nusselt number value at the left wall (i.e., Nu_1) increases with increasing the Forchheimer number. On the other hand, the Nusselt number values at the right wall (i.e., Nu_2) are different and decrease as F increases. This behavior of the heat transfer coefficients (i.e., Nu) can be explained by the fluid temperature distribution.

The variations of the convective heat transfer coefficient at the channel walls with mixed convection parameter (i.e., GR) are summarized in Table 1. As observed from

this table, the effect of GR values on the Nusselt number values is similar to that on the temperature distribution.

The comparison of the results predicted for dimensionless velocity and temperature distributions by the perturbation and numerical methods is shown in Figure 8. As observed from this figure, there is fair agreement between the predicted results of the perturbation and the numerical methods. It is worth mentioning that with increasing the perturbation parameter, the difference between the predictions of the numerical and approximate analytical methods increases.

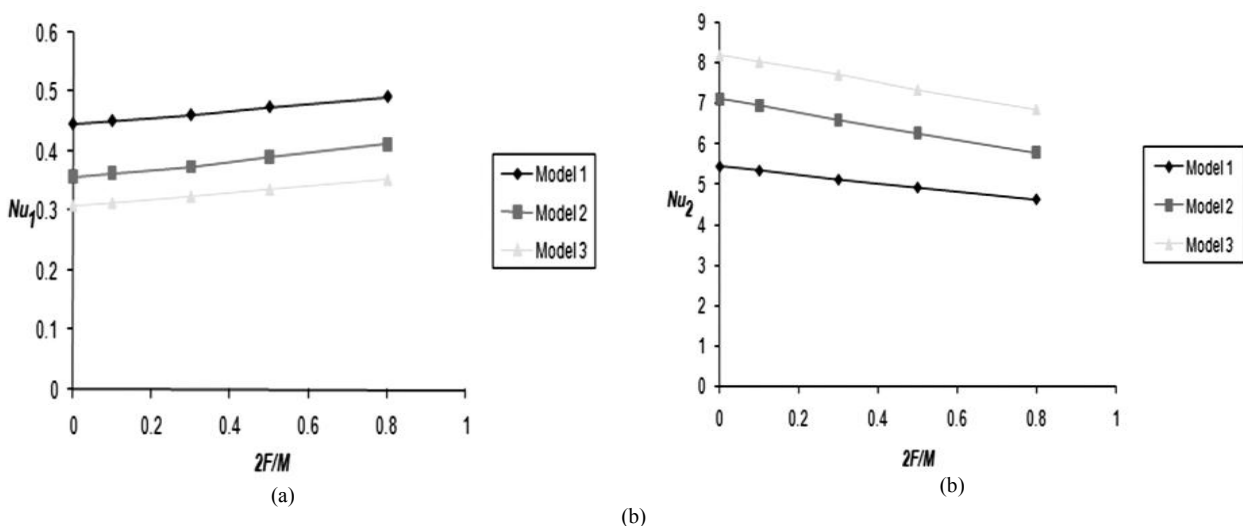


Figure 7. Effects of Forchheimer number on (a) Nu_1 and (b) Nu_2

Table 1. Values of Nu_1 and Nu_2 for different values of GR

	$Br = 0.02, \gamma = 0.01$					
	Model 1		Model 2		Model 3	
GR	Nu_1	Nu_2	Nu_1	Nu_2	Nu_1	Nu_2
0	0.666	2	0.666	2	0.666	2
5	0.570859	3.177477	0.523647	3.620565	0.493911	3.881658
10	0.263796	12.61807	0.167147	19.48521	0.128636	24.14251
15	0.073209	57.69149	0.034801	106.8312	0.023925	142.4125
20	0.020247	219.5822	0.008492	445.9768	0.005589	615.7845

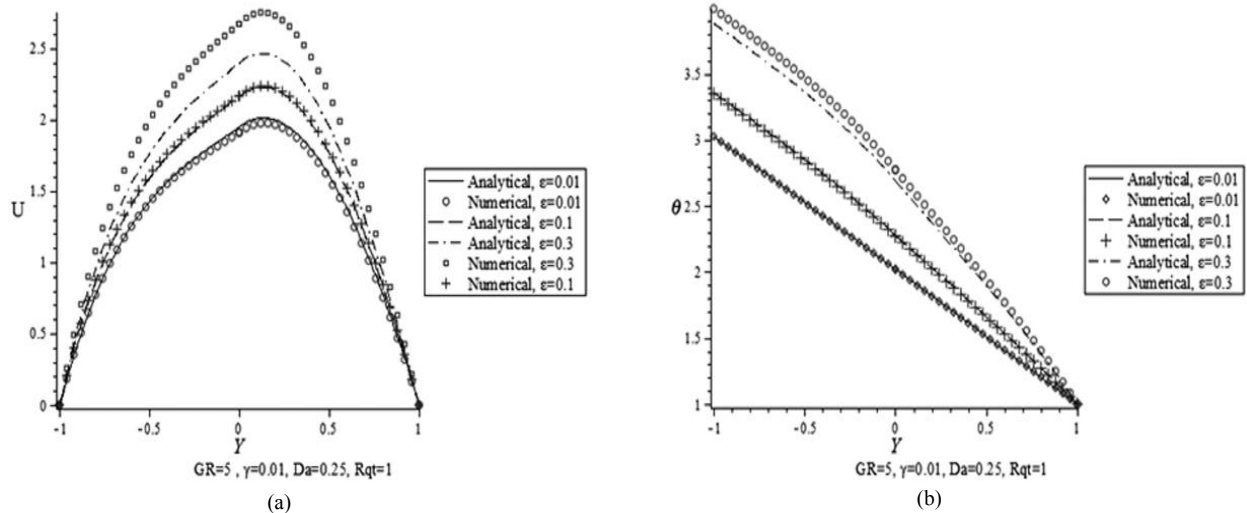


Figure 8. Comparison of numerical and analytical results (a) velocity profile and (b) temperature profile

CONCLUSION

Approximate analytical and numerical solutions were obtained for the problem of mixed convection in channels partially filled with a porous medium taking into account the effect of viscous dissipation and the inertial force. The problem of combined forced and natural convection with hybrid thermal boundary conditions was solved by two parameter perturbation and numerical methods. Three different viscous dissipation models were considered to account for the viscous heating. The velocity and temperature distributions for both the porous and viscous regions and the Nusselt number values were also obtained in terms of Grashof, Reynolds, Forchheimer, Brinkman, and Darcy numbers. Moreover, it was found that the influence of type of viscous dissipation model on the velocity and temperature distributions is profound and increasing the values of porous parameter and Forchheimer drag term reduces the flow in the channel.

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NOMENCLATURE

A = negative of applied pressure gradient (dp/dx)
 b = thermal expansion coefficient ratio (β_2/β_1)
 Br = Brinkman number
 C_p = specific heat at constant pressure
 C_F = inertial coefficient
 D_a = Darcy number based on h_1
 F = Forchheimer number
 g = gravitational acceleration
 Gr = Grashof number
 GR = buoyancy force to pressure gradient ratio
 h = width ratio (h_2/h_1)
 h_1 = porous region width
 h_2 = viscous region width
 K = permeability of porous media

k = thermal conductivity of fluid

m = viscosity ratio (μ_1/μ_2)

$M = \mu_{eff}/\mu_1$

n = density ratio (ρ_2/ρ_1)

p = pressure

q_w = wall heat flux

Re = Reynolds number

T = temperature

T_w = prescribed boundary temperature

u = velocity

Greek Symbols

α = thermal diffusivity

β = thermal expansion coefficient

ε = dimensionless parameter

γ = dimensionless parameter

κ = thermal conductivity ratio (k_1/k_2)

μ = viscosity

μ_{eff} = effective viscosity

ν = kinematic viscosity

θ = dimensionless temperature

ρ = density

σ = porous parameter

Subscripts

1 & 2 = Reference quantities for region I and II, respectively

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