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The Effect of Permeability Contrast in Percolation Reservoir Models on the Breakthrough Time Distribution

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Abstract
In waterflooding process, the time for breakthrough of injecting fluid into a production well is of great importance. Predicting this time helps in designing reservoir development plan. Due to uncertainties in reservoir characterization, estimating the breakthrough is not easy, so alternative methods to estimate quickly the breakthrough time is useful. The percolation method uses limited available reservoir data to predict the breakthrough time distribution, and it may be used for engineering applications. However, implementation of this to real reservoirs requires some adjustments. The aim of this study is to show how percolation approach can be used to real problems. In particular, the effects of permeability contrast between the reservoir and non-reservoir parts in the model are investigated. In order to use the breakthrough scaling function to more realistic reservoir models, a dimensionless breakthrough time was used. The analysis of the breakthrough time of models with zero permeability background ($t_{k=0}$) and such time for the case of non-zero permeability background ($t_{k=\alpha k}$) shows a linear dependency which can be used to find breakthrough time distribution. Hence, this correction extends the applicability of the percolation method for predicting breakthrough time when permeability of the system background is not zero.

Keywords: Porous Medium, Percolation Theory, Breakthrough Time, Non Zero Permeability, Permeability Contrast.

Introduction
The study of fluid flow, i.e. liquid or gas through a porous medium, in either pore scale or very large scale is important in many fields including hydrology and petroleum engineering. The movement of a fluid in a porous medium is controlled by the involving forces and the spatial distribution of heterogeneity in the medium which may form a critical path (so called minimum path) [1]. This path controls the breakthrough time, i.e. the minimum time for passage of a fluid, between two points in the medium. In many practical cases, the distribution of the heterogeneity itself is the main physics for the fluid flow in porous media. The flow characteristics of such systems are mainly controlled by the continuity of the conductivity contrasts. An example of such heterogeneous systems is the geological formation which consists of a mixture of good sandstones with high permeability and poorer siltstones, mudstones, and shales with negligible permeability.

In this paper, percolation theory approach is used as an appropriate model of connectivity and dynamics in complex geometries [2]. This approach provides some simple power laws from which likely outcomes can be predicted by simple algebraic transformations [3]. This approach is described briefly by starting with or explaining a very simple porous medium. The medium is assumed to be a lattice with a fraction p of randomly occupied (or permeable) sites and the fraction 1-p of non permeable sites. Clusters are formed when neighbouring sites are occupied. The properties of the medium are related to the number and the size of these clusters, and they vary as a function of this occupancy probability p. As the fraction p is increased, the clusters grow in size. At one particular p (called percolation threshold, p_c) for the first time, one large cluster can connect the two sides of the medium (so called spanning or percolating cluster). However, there are also other small clusters which get absorbed to the percolating cluster as p further increases. Therefore, below the threshold ($p < p_c$) the probability of existing a globally connected cluster is zero. For infinite size medium, above the threshold ($p > p_c$), the probability of having an infinite cluster is equal to one although for finite system, this is less than 1. Due to the random structure of the medium and consequently, the existence of many states for the flow within the medium, the threshold point must be estimated in a statistical approach. To get a reasonable statistics, a large number of possible realizations of the system must be generated

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and from them an average and standard deviation of percolation threshold can be inferred. Previous studies have shown that the threshold depends on the system characteristics (e.g., the system dimension and occupation probability) [1,3]. As an example, when injecting a fluid into a porous medium for the pressure maintenance purposes in secondary recovery processes, the larger the volume of the injected fluid is, the greater the saturation of the injected fluid in the porous medium is, and the greater possibility of connection between the two sides of the system is [4].

Quantitative estimation of percolation properties e.g., the threshold value and the estimation of inter-well connectivity through permeable regions within the medium can be obtained by using percolation theory concepts [5]. Different varieties of percolation enable us to not only study the static property of the heterogeneous media but also be able to evaluate the dynamic behavior of such system. For example, the fluid tends to pass through the least resistance paths and so narrower paths with lower permeability may not be swept. Also, due to the pressure change or fluid saturation, a phase may be trapped and blocked. In such a case, the flow tries to alternative pathway and so tortuous structure can be obtained. In dynamic percolation approach, not only the percolation theory concepts but also the appropriate fluid flow equations related to the physics of the problem (e.g., mass transfer equations) can be used [6].

From field-scale application of percolation theory, it is possible to estimate the breakthrough time of injected fluid, a suitable well spacing, the fraction of sands connected between two wells and the post breakthrough behavior, e.g. rate of oil production decline [7].

The regular percolation models can be formed on the lattice of sites (e.g., square, triangle, honeycomb) or network of bonds. The bond network model consists of pores and throats [1]. The more applicable network model used in petroleum engineering made of sandbodies with different shape, diameter and lengths that are connected together in a form of a network of pores and throats. Examples of network models are shown in Figure 1. In such continuum models, the occupancy p is defined as the area fraction covered by the good sands (discounting the overlapping).

The aim of this study is to determine the breakthrough time of an injected fluid into the production well at field scale by using percolation scaling relations. However, as the complexity of the reservoir scale makes the simple assumptions used in the primary percolation scaling relations questionable; hence, in this study, a method to extend the applicability of the predictions of the percolation method to the actual behavior of hydrocarbon reservoirs is presented by us. In this study, the main variable is breakthrough time. The strategy of this research is as follow: First, simple percolation models with its limitations are considered by us, and the first estimation of breakthrough time is obtained. Afterwards, to improve the predictions, the related reservoir variables are analyzed, and their effects are considered into the percolation scaling relations in an appropriate way. To justify the methodology, comparisons of the results of the improved scaling relations with outputs from commercial flow simulator are made based on the execution of a large number of flow simulations.

Theoretical Backgrounds

The application of dynamic percolation to model flow in a porous medium started in 1980s. In primary studies, the porous medium is first saturated by one phase, and it was displaced by an injection phase through the pore network where during this process some images of the clusters of the injected phase formed were taken [8,9]. However, the trapping effect of the displaced phase was neglected. In 1983, the effects of trapping are considered by Wilkinson and Willemsen [10], and they were shown for the first-time so-called invasion percolation. This was a quasi-state displacement used to model immiscible two phase displacement where the capillary forces are critical. Wilkinson and Willemsen's results were based on the simulation results, and the scaling relation presented was with no reasonable justification. However, this approach was developed rapidly, and its mathematical reasoning was found [11]. Much of the literature on dynamic percolation follows similar analysis at pore scales. A power law relation for the system global dynamic properties in dynamic percolation has been reported by most of researchers [12-13]. More details for the dynamics of fluid flow in percolating systems can be found in Blunt [14] or King and Masihi [3].

An important feature of porous media, which affects its dynamic properties, is the level of heterogeneity and the scale dependency of the dynamic properties of porous media. There are some publications which show the scale dependency of the dynamic properties of porous media. Recently, has studied the effect of sample size on hydraulic characteristics of porous media has been studied by Ghanbarian in 2022, and their scale-dependent properties have been investigated [15,16].

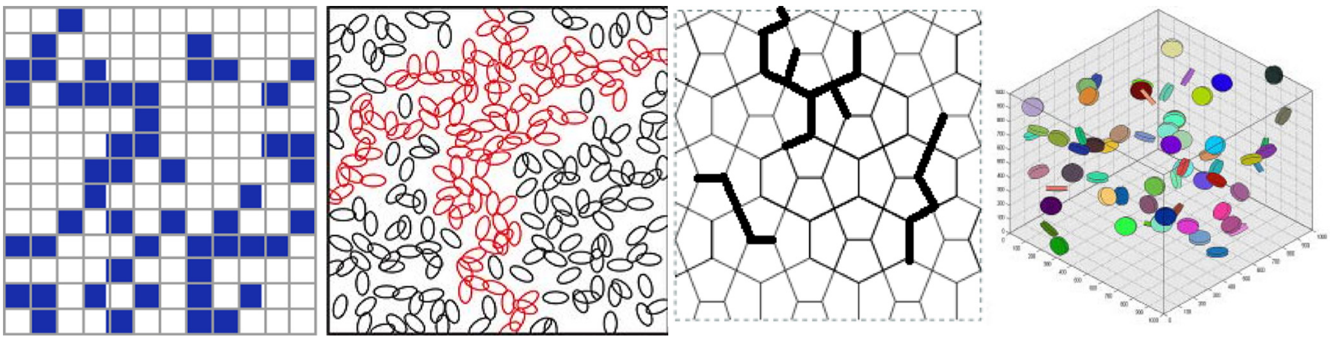


Fig. 1 Types of percolation models: a) lattice of square sites (2D), b) overlapping ellipses (2D), c) network of bonds(2D) and d) overlapping ellipsoids (3D).

Recently, has studied the effect of sample size on hydraulic characteristics of porous media has been studied by Ghanbarian and their scale-dependent properties have been investigated [15,16]. In particular, a combination of percolation theory concepts along with finite-size effect by considering the following items: porosity, critical porosity, critical scaling exponent, a fundamental length scale, and system length has been used to develop a theoretical relationship for permeability estimation. Moreover, has successfully been evaluated on various types of porous media.

To predict the rescaled frequency-dependent permeability and evaluate its universality, the known effective-medium approximation (EMA) for porous media, characterized by a pore-size or pore-conductance distribution, has been used by Sahimi [17]. This is useful when the heterogeneity of the pore space is not very strong and the critical-path analysis when the pore space is highly heterogeneous. The results for the frequency-dependent permeability can be extended to the other properties including: (1) electrical conductivity, (2) the formation factor, and (3) the diffusion and dispersion coefficients of porous media.

However, there is a few published reports on the petroleum engineering applications of percolation method (see the reference [3, 14] for giving further details). Most of these focused on the pore scale studies. At this scale, the imaging capabilities have shown a nearly random structure for the pore distribution (or permeability and fluid saturation distributions), so there is a need for developing appropriate scaling equations to characterized the dynamic behavior of these mediums. Also, some assumptions used in percolation theory limits the application of it for realistic field models. For example, the scaling ansatz of percolation properties such as breakthrough time was originally developed by using random Walk and Monte Carlo simulation on regular lattices. However, more realistic porous media can be made of overlapping sands, and the physics of two phase displacement is needed. The connectivity and conductivity between point-shaped locations were estimated by Soltani and Sadeghnejad [18], which it represents the point-to-point connection in a 2-D areal cross sectional map of a 3-D porous medium containing vertical wells. Their results showed a significant difference between the connectivity of connected cluster formed between two lines located at the opposite sides of a 2-D porous medium (i.e., injectors and producers in the petroleum terminology) and the connectivity of the connected cluster formed between two points. Monte Carlo simulation on the fracture models with a distribution of fracture length was used to study the effect of fracture lengths on the effective permeability, the mean porosity, the excluded volume (area), and the percolation threshold of the network by Hamzehpour et al. [19].

Moreover, to quantify the connectivity, hydraulic conductivity, and breakthrough time behavior between an injector and a producer within systems with the spatial distribution of geological heterogeneities, percolation theory was used by Sadeghnejad et al. [20]. In this work the heterogeneities was by overlapping sands. It is shown that scaling ansatz of percolation breakthrough time, for example, was similar to those developed on porous media made of regular lattices.

To predict breakthrough and post breakthrough behaviour, percolation theory was used by Sadeghnejad and Masihi. Afterwards, a comparison was made between the results obtained from the percolation theory, which was applied, and numerical flow simulation results which was used by the Burgan formation dataset of Norouz offshore oilfield in the south of Iran [21].

Materials and Methods

Scaling for the Breakthrough Time Distribution

First, in 2000, the percolation based scaling equation has been studied for a system size (L) and the occupancy (p) with a table of general values for the coefficients and exponents by Andrade et al. [22]. sLater, in 2000, a scaling equation for the breakthrough time of an injected fluid in a producer with a distance, r, located in the system of size L has been developed by King et al. [23]. In their study, a power law scaling equation (Equation 1) with the fractal dimension of the backbone fraction dB and an exponent β was used to predict the volume of oil production after the breakthrough time. The numerical value for the exponent β based on the simulation results was 0.63± 0.0008 [23] (King, Buldyrev et al.).

$$V\ (t \to \infty) \sim (\frac{r^{d_B}}{t})^{\beta} \tag{1}$$

In tis study, the following scaling equation (Equation 2) reported for the breakthrough time estimation in reservoirs under water injection with variables summarized in Table 1 [23] (King, Buldyrev et al.). The occupancy p in percolation terminology is equivalent to the net to gross (NtG) in petroleum engineering. As said before, the threshold value (p_c) depends on types of percolation network used in the model. It is also emphasized that the exponents given in Equation 2 are independent, and they have some relations with the fractal dimension of the backbone and the critical exponents as reported in Reference [24].

$$P(t_{br} \mid r, L, p) \sim \frac{1}{r^{d_t}} (\frac{t_{br}}{r^{d_t}})^{-g_t{}'} \exp(-a(\frac{t_{br}}{r^{d_t}})^{-\varphi}) \\ \exp(-b(\frac{t_{br}}{L^{d_t}})^{\psi}) \exp(\frac{-ct_{br}}{\mid p-p_c \mid^{-\nu d_t}}) \tag{2}$$

In a simple 2D model, the reservoir is made of randomly distributed sands (as square objects) with possibility of overlapping to each other. The numerical values for the variables and constants in Equation 2 obtained from simulations of 2D and 3D problems are given in Table 2. The scaling equation (Equation 2) for the breakthrough time has originally developed based on some simple assumptions with some finite geometric variables. Thus, the application of this scaling equation for the real field where injected fluid (e.g., water) has different properties than displaced fluid (e.g., oil) or the contrast in the permeability of reservoir rock may not be critical, may be with caution. In particular, the effect of the difference between the absolute permeability of the permeable regions and non-permeable parts is not considered in Equation 2.

Table 1 Definition of variables in the scaling equation (Equation 2) [22, 23].

$P(t_{br},\| r, L, p)$	Breakthrough time distribution t_{br} for a 2D system of size L with occupancy p and well spacing r
t_{br}	Dimensionless Breakthrough time
L	Dimensionless length, i.e., L=system size/sand size
p_c	Threshold of overlapping sands in 2D (p_c =0.668)
r	Dimensionless well spacing, r=well spacing/sand size
p	Occupancy probability or Net-to-gross
v	Correlation length critical exponent (3/4 in 2D)
$a, b, c, d_t, g_t, \varphi, \psi$	Coefficients and exponents with numerical values in **Table 2**

Table 2 The numerical values for the variables and constants in scaling equation (Equation 2) [7, 22, 23].

Parameters and exponents	2D	3D
a	1.1	2.5
b	5	2.3
c	(if p<pc) 1.6 (if p>pc) 2.6	(if p<pc) 2.9 --
d_t	1.33±0.05	1.45±0.1
g	1.9±0.1	2.1±0.1
φ	3	1.6
ψ	3	2

Therefore, considering the effect of rock variation in the reservoir and more realistic fluid properties in the scaling equation of the breakthrough improves the physics of the two phase displacement and extends applicability of the scaling equation to real fields [3].
King et al. used a flow-based dimensionless time equation to incorporate the effects of flow geometry in the scaling Equation 2 [7]. Their proposed two-dimensional times were based on the Darcy relation for linear or radial flow as follows [23]:

$$t_s^{radial} = \frac{c_2 \phi p \mu r_s^2 \ln(r_s / r_w)}{K \Delta P (r / r_s)} \qquad (3)$$

$$t_s^{linear} = \frac{c_1 \phi p \mu r_s^2}{K \Delta P (r / r_s)} \qquad (4)$$

where ts is the time takes for a fluid to pass through a permeable sand of size rs. To determine this time, Darcy equation has been used and assumed a number of sand blocks located between a pair of production and injection well. For flow in radial geometry this time become,

$$t_s = \frac{r r_s \mu \ln(\frac{r}{r_w})}{1.127 * 5.615 * k_s (p_{Injection} - P_{Production})} \qquad (5)$$

A similar approach can be used to find this reference time in linear flow geometry [3, 23, 25]. The time variables in Equation 2 are then normalized with this reference time, as seen in Equation 5.

Model Buildup

To build the reservoir model, it is first necessary to have a static reservoir model with percolation structure. A summary of assumptions used for building static models is given in Table 3.

Table 3 Assumptions used to build static models

	Variable	Value	Unit
1	φ	Reservoir: ϕ=0.25 Non reservoir: ϕ=0	fraction
1	K	Reservoir: from Karman-Cozny relationship k=10^4 $\phi^3/(1-\phi^2)$ Non reservoir: K=0	Darcy
3	NtG	1	-
4	Grid	Number=10^3 cells Size= 200×200×1	Numbersm^3
5	Kr	Linear curves (or x-shape curves) with no residual saturations	
6	Scenario	Production well in cell (50,50), Injection well in cell (150,150), rate of 50 bbl/d for voidage ratio=1	

The net to gross considered in the model is NtG= 1 for the whole reservoir, and the reservoir and non-reservoir sections were separated by differences in porosity and permeability values. These 2D models with the aforementioned data have been made automatically using coding in the software environment using the workflow section. This modeling was obtained by performing two independent simulation loops. In the first loop, different realizations with normal distribution of permeable sands distributed randomly in non-permeable medium were created. At this stage, a certain number of models were built.
The production and injection wells were located relatively far from each other and also far from the boundaries in order to have clusters of connected sands connected through these two wells at occupancy probability near to the percolation threshold. It is emphasized that locating the wells on the boundaries may need a high occupancy probability (net-to gross) to have connected cluster of sands which includes the two wells. Due to the random distribution of the sands, there may not be connection between the injection and production wells in some of the created models. Therefore, a second loop was defined to evaluate the connectivity status in the system. In this loop, the models with no connection of sands between the production and injection wells were removed from the

statistical population. Afterwards, for the rest of the models, the petrophysical properties have been assigned. For each these static models, porosity value of, for example, 25% was assigned to the permeable cells, and for the other non-permeable cells zero porosity value was considered. The amount of permeability of each cell depends on the porosity through the power law $k=10^4\ \phi^3/(1-\phi^2)$. The NtG value for all the cells was equal to one. After removing the undesirable models with no connectivity, if the number of realizations is not statistically suitable, the process of generating models will be repeated. The minimum number of generated desired models for the base case is set to 150 models. It is emphasized that at lower occupancy, p (e.g., p = 0.5), the probability of generating models with no connectivity is higher, so a large number of generated models which has no connectivity would be removed. For this reason, a significant time (e.g., about 500 hrs) with a system: Intel (R) Core (TM) i7-10510U CPU @ 2.3 GHz) is needed to get a reasonable statistics. However, for higher p values than the threshold (e.g., *p* = 0.8), the probability of connectivity is high and almost all the generated models are accepted, so there may not need to repeat the process of model generation.

In Figure 2, realizations at 5 different occupancy probabilities with finite non connected clusters which have been illustrated in different colors as well as the spanning cluster (if existed) are shown. As seen, the number of finite clusters decreases as the occupancy probability p increases.

In Figure 3, the porosity and permeability models are shown for two case: p = 50% and p = 70%. In these models, the porosities of sands (permeable units) and shales (impermeable background) are 25% and 0.0025%, respectively.

Similarly, the permeability of sands and shales are equal to 1 and 100 millibars, respectively. As seen, an increase in the average porosity and permeability of sands is obtained by increasing the occupation *p*.

Dynamic Simulation

To determine a sufficient number of realizations for later statistical analysis of the breakthrough time, the mean breakthrough time against the number of realizations has been investigated.

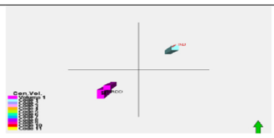
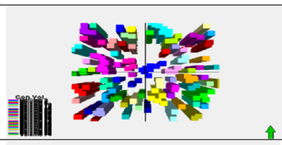
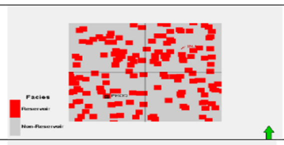
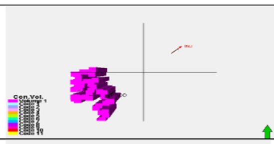
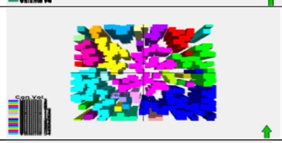
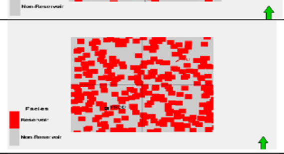
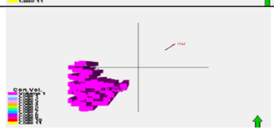
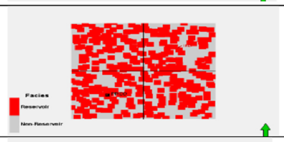
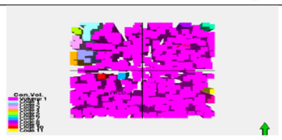
p	No. of clusters	Clusters related to wells	clusters	occupations	comment
30%	71				No connection between the wells many small size clusters
50%	34				No connection between the wells and larger finite clusters
60%	25				No connection between the wells and larger finite clusters
70%	15				Connection between the wells and few small finite clusters
80%	12				Connection between the wells and few small finite clusters

Fig. 2 Examples of clusters formed and their connections for some static models with different occupancy, *p*.

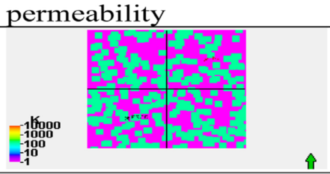
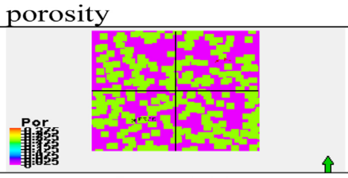
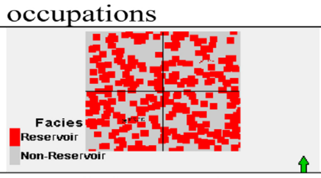
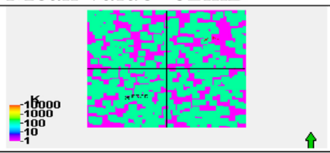
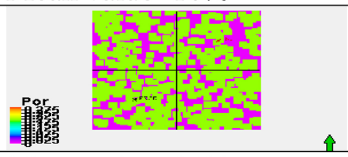
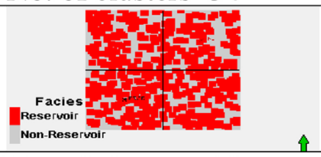
p	permeability	porosity	occupations
50%			
	Mean value=62mD	Mean value=10%	No. of clusters=34
70%			
	Mean value=87mD	Mean value=14%	No. of clusters=15

Fig. 3 Realizations of the permeability and porosity models at two different occupation probability *p*=0.5 and 0.7

For the reservoir base model, the average breakthrough times were determined in the range of 2 to 5 years. To get reasonable simulation results, it is need to set an appropriate simulation time steps to get reports for later analysis (e.g., analysis of breakthrough times). Considering a small value will results in huge simulations files. On the other hand, taking a high value will give the same breakthrough time, for example, for many reservoir models/realizations. Therefore, after a sensitivity study, an appropriate time step is set to 5 days. This can be also considered as a tolerance in breakthrough time calculations. Hence, the tolerance assumed for the breakthrough time was considered to be 5 days. Therefore, the number of realizations are increased until the variations around the average breakthrough time fall into the range ±5 days.

Analysis of the number of realizations at each system size and occupancy (p) value shows that the number of realizations increase near the percolation threshold where the models are very heterogeneous. However, away from the threshold (either below or above the threshold), a smaller number of realizations are required. Examples of the number of required realizations at occupancy values of p = 0.7 (close to the threshold i.e., p_c=0.688), 0.55 and 0.85 are respectively 124, 92 and 68. For simulation purposes, a production well in the cell (50, 50) and an injection well in the cell (150, 150) were defined as a base scenario.

The rate of 50 barrels per day was considered to maintain the voidage ratio of 1. The other reservoir rock and fluid data used to perform the flow simulations is presented in Table 4.

Table 4 Reservoir rock and fluid data considered for the flow simulations.

Variable	Value	Unit	Equation
P_i	4000	psi	-
T_i	170	f	-
S_{wi}	S_{water}=0, S_{gas}=0, S_{oil}=100	%	-
R_s	210	scf/stb	Lasater (1958)
salinity	200000	ppm	McCain (1990)
B_o	1.13	bbl/stb	-
P_b	1120	psi	-
µ	0.8 – 1.43	cP	-
P_c	0	psi	-

As we need to run flow simulations on large number of realizations, this step was performed by coding in the software environment using the workflows section which defines a case for each scenario. Afterwards, the pressure solver software for Black oil model solves the flow equations in the software environment.

After simulation of each scenario, the software execution screen closes and the process of creating a new realization and running the simulation on it starts, and this will be repeated. The numerical simulation will be continued until the water cut in production well reaches the pre- determined value. Afterwards, the results are read from the simulation output file and recorded consecutively in a .txt file. As the initial water saturation in the model was assumed to be Swi=0, the water cut until the injection water reaches the production well is zero. Hence, any positive water cut (which is assumed to be $S_w=10^{-6}$ in this study) means that the injection of water breakthrough into the production well occurs.

It is emphasized that the simulation run time for models at higher occupancy p (e.g., p = 0.75 or higher) is longer due to the fact that the flow fills a larger volume along its path from the production well to the injection well. Hence, the flow simulations in all realizations requires a relatively long time (for example, 250 hrs for simulation of a model at an occupancy p value using a system: Intel (R) Core (TM) i7-10510U CPU @ 2.3GHz).

In the next step, the breakthrough time values obtained from the simulation were analyzed the MATLAB environment by plotting the histogram of the breakthrough time values. Also, the percolation scaling equation (Equation 2) for each model was used to determine the breakthrough time probability distribution. Afterwards, the histogram of the breakthrough time values obtained from flow simulations was compared qualitatively with probability distribution dawn from percolation scaling. Also, Q-Q plot analysis was used to compare these two set results. Using the R2 error of the Q-Q plots and the differences in the histogram plots, the accuracy of the percolation scaling law was evaluated.

Results and Discussion

Simulation Results

In this section, on the effect of the permeability contrast between the sands and the background in the medium (i.e., shale) has been focused. By permeability, we mean the absolute permeability; also, the two phase relative permeability curves are assumed to be linear (so called X-type) in all scenarios, and the residual saturations are equal to zero. This assumption ignores the effects of two phase interactions, and so we can only focus on the impact of permeability contrast in the medium. Also, this is in line with the assumption of passive tracer movement (in random walk model) used initially to develop the scaling ansatz of percolation breakthrough time (Equation 2).

In this section, two sets of permeability models are considered. The permeability of the reservoir parts (i.e., sands) is assumed to be fixed e.g., equal to k; while the permeability of non reservoir parts (i.e., shales) was assumed to be either zero (in the first set of reservoir models) and a small but non-zero value (such as 0.001K) in the second set of models. To implement this in the software, two types of facies (or rock type) were defined. Afterwards, porosity values for each of these two facies were assigned (e.g., ϕ=25% and 2.5%), and by using permeability-porosity relation ($k=10^4\ \phi^3/(1-\phi^2)$), the permeability of these two facies were determined. This is known Karman-Cozny relationship between porosity and permeability, which it was also used in previous works [26].

The simulation in almost all scenarios proceeds at occupancy probability below the threshold to some values well above the threshold (e.g., p = 50% to p = 80%). As said before, a tolerance of ±5 days for the average breakthrough time was asumed to find the reasonable number of realization in order to get reliable statistics of the breakthrough time. To illustrate this, the plot of the average breakthrough time against the number of realizations for the model with an occupancy p= 55% is shown in Figure 4.

As can be seen, the number of necessary realizations were respectively 45 and 38 for models with background permeability of $k_{non\ res}$=0.001k and $k_{non\ res}$=0. This shows that models with the non zero background permeability needs more realizations.

It is emphasized that similar analysis for estimation of the appropriate number of realizations for the other cases have been done. It is emphasized that the value of the background permeability in the model is three orders of magnitude lower than the permeability of the reservoir part. This significant contrast creates a reasonable time scale difference for the flow movement within each part. Comparison of the histograms of breakthrough time in the simulation models with occupancy *p*= 55% to 75% for two cases with the background permeability of $k_{non\ res}$=0.001k and $k_{non\ res}$=0 is shown in Figure 5. As seen, the simulation results in a higher breakthrough time in models with the zero background permeability $k_{non\ res}$=0.

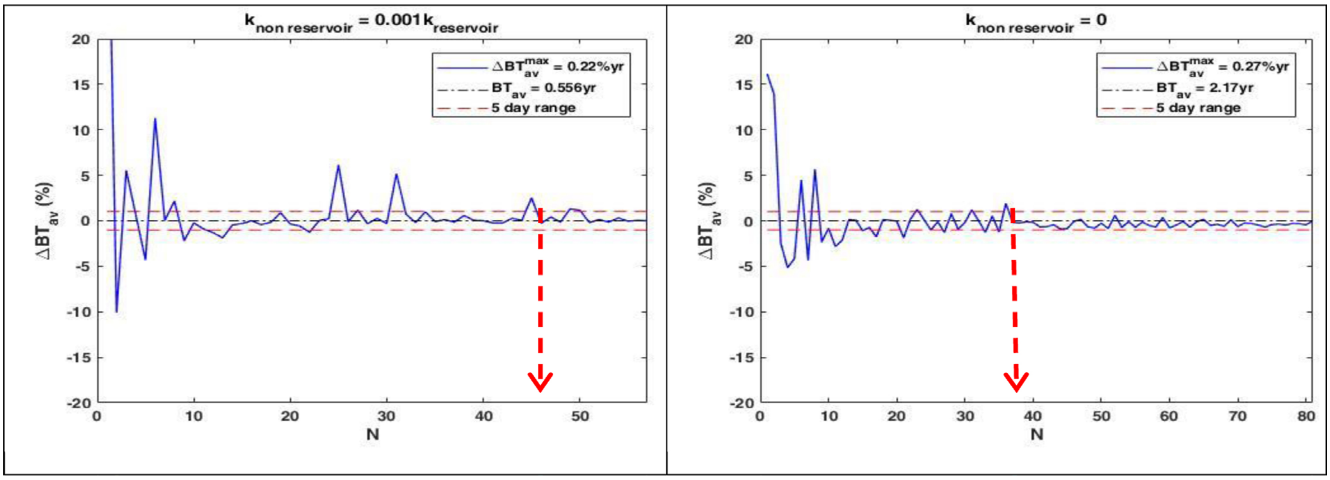


Fig. 4 Variations of the change in the mean breakthrough time (ΔBT_{av}) against the number of realizations (N) for the simulation models with occupancy *p*= 55% for two cases where the absolute permeability of the background (shale) is 0.001k (left) and zero (right) where the reservoir permeability is k=5 Darcy..

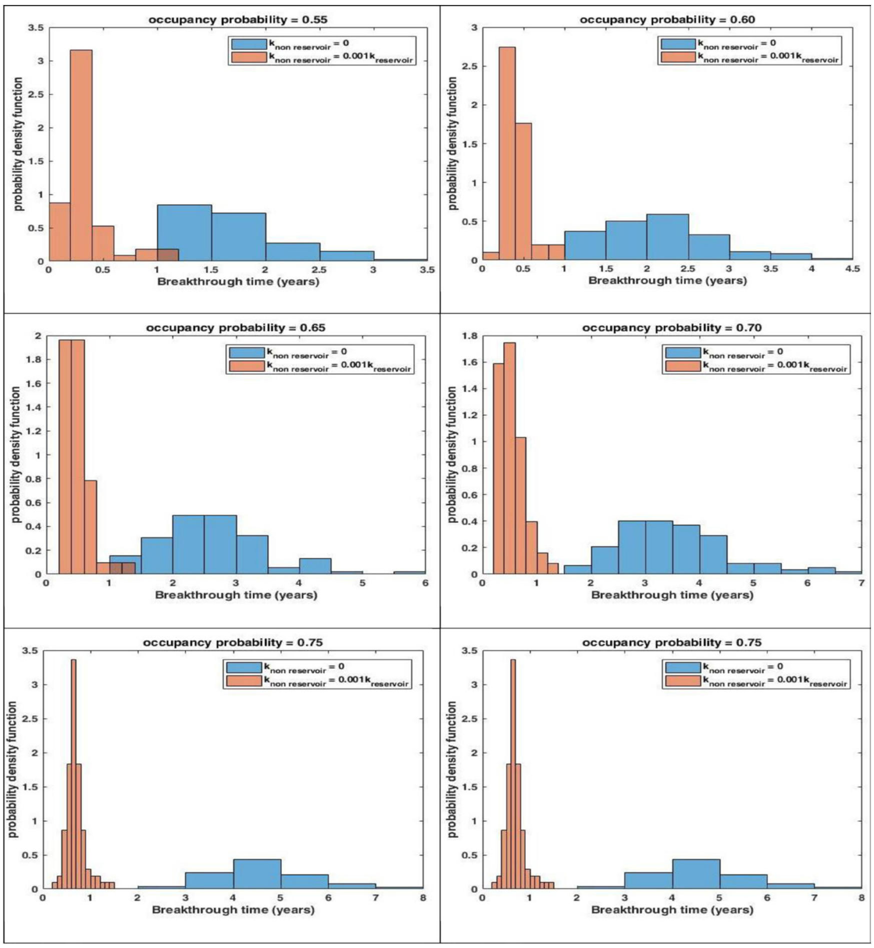


Fig. 5 Illustration of the histograms of breakthrough time in the simulation models with occupancy p= 55% to 75% for two cases with the non zero permeability of the background i.e., $k_{non\ res}$=α.k and $k_{non\ res}$=0 where k is the sand permeability and α =0.001.

Also, this figure shows a longer time needed for the breakthrough of injected fluid in the models with a higher occupancy probability p. As discussed in Shokrollahzadeh et al studies which have been carried out [25], the correct form of Equation 2 to be applicable in real problems is when we use the time variable in Equation 2 in a dimensionless form by normalizing with a characteristic time scale. For example, as can be seen in Figure 6, the flow geometry is changed from linear to radial flow geometry when occupancy increases from the values near the percolation threshold to some value close to p=1. As emphasized by Shokrollahzadeh et al, this effect of flow regime change which appears with a higher volume of injected fluid and a longer breakthrough time could be incorporated in the characteristics time scale used in Equation 2 [25].

More details results of the breakthrough time and the number of realizations are summarized in Table 5.

Table 5 A summary for the results of the breakthrough time and the number of realizations shown in figure 5 for two cases with the non-zero permeability of the background i.e., $k_{non\ res}$=α.k and $k_{non\ res}$=0 where k is the sand permeability and α =0.001.

p	Description	Realizations	Ave breakthrough time, year
0.55	$K_{non\text{-}reservoir} = 0$	81	2.17
	$K_{non\text{-}reservoir} = 0.001K_{reservoir}$	57	0.556
0.6	$K_{non\text{-}reservoir} = 0$	92	2.37
	$K_{non\text{-}reservoir} = 0.001K_{reservoir}$	51	0.303
0.65	$K_{non\text{-}reservoir} = 0$	105	2.77
	$K_{non\text{-}reservoir} = 0.001K_{reservoir}$	51	0.436
0.7	$K_{non\text{-}reservoir} = 0$	124	3.56
	$K_{non\text{-}reservoir} = 0.001K_{reservoir}$	63	0.629
0.75	$K_{non\text{-}reservoir} = 0$	93	4.99
	$K_{non\text{-}reservoir} = 0.001K_{reservoir}$	51	0.619
0.8	$K_{non\text{-}reservoir} = 0$	92	5.84
	$K_{non\text{-}reservoir} = 0.001K_{reservoir}$	51	0.852

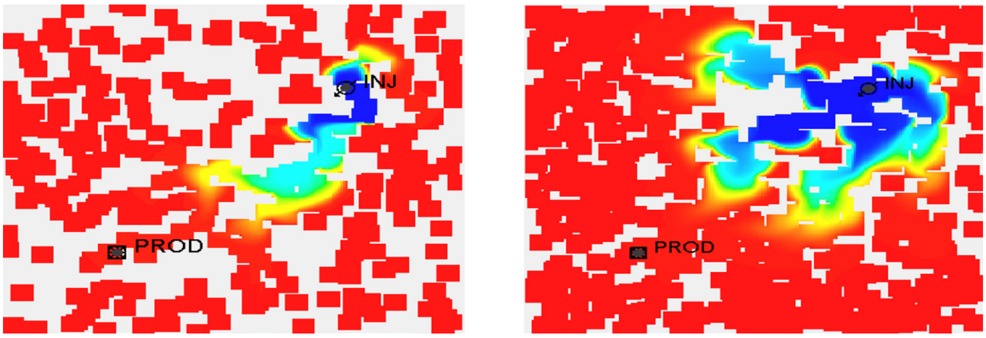


Fig. 6 Illustration of the effect of flow regime change from linear to radial as occupancy increases from: (left) p=0.5 to (right) p=0.8.

Figure 7 shows the oil volume distribution (defined by the product of oil saturation and reservoir porosity) for two cases with the background permeability of $k_{non\ res}$=0.001k and $k_{non\ res}$=0 to evaluate the impact of the permeability of the background (i.e., shale) in the reservoir. It is worth mentioning that the plot of oil saturation only was not able to characterize its variation. Hence, the product of porosity and oil saturation was used in Figure 7. The porosity and permeability of the non-reservoir parts in Figure 7 (right) is zero while these are 2.5% and 5 Darcy for model shown in Figure 7 (left). Also, as shown in Figure 7, the fluid flow can find other new paths for the case with the non zero background permeability which results in shorter breakthrough time.

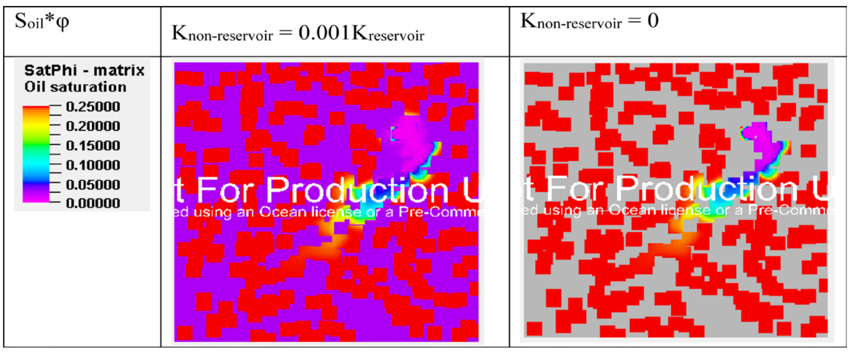


Fig. 7 Illustration of oil volume variation (shown by $S_o \times \phi$) for two cases with the non zero permeability of the background i.e., $k_{non\ res}=\alpha.k_{res}$ where k is the sand permeability: (left) α =0.001 and (right) α =0 or $k_{non\ res}$=0 to evaluate the impact of the permeability of the background (i.e., shale) in the reservoir.

The comparison of the breakthrough time of models with zero permeability background ($t_{k=0}$) and the breakthrough time of models with non-zero permeability background k=0.001 md ($t_{k\neq0}$) for reservoir models at occupancy probability around and above the threshold is shown in Figure 8. As can be seen for the results of breakthrough times of this case with permeability of k=0.001mD (see Figure 8), a positive correlation between these two times has been found out clearly. It is noted that the permeability of the background in this case is $k_{non\ res}=\alpha.k_{res}$ where α=0.001. Although this clearly shows the significant difference in the results compared with the breakthrough results of models with zero background permeability value, the generalizing this needs more reservoir model cases (with different α values) to be made and run flow simulations.

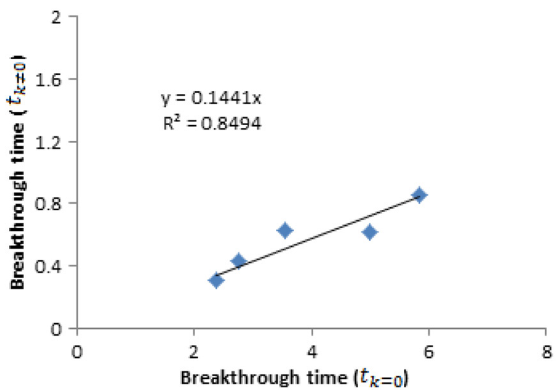


Fig. 8 The comparison of the breakthrough time of models with zero permeability background ($t_{k=0}$) and the breakthrough time of models with non-zero permeability background ($t_{k=\alpha k}$) which shows a significant trend.

The analysis of the results shown in Figure 8 for the case α=0.001 indicates the following correction term for the reference time (i.e., Equation 6) that may be used in the breakthrough time scaling function to account for the non-zero permeability of the background in the system.

$$t_{k\neq0}=\alpha^m\, t_{k=0} \quad (6)$$

where the numerical value of the exponent m based on the breakthrough time shown in Figure 8, and α value equals 0.001 is m~0.3.

Conclusions

In this study, percolation theory approach has been used to predict the breakthrough time of injected fluid in oil reservoirs with non zero permeability of the background in the system. In particular, the scaling relation for the breakthrough time distribution based on the percolation concepts was used. In this study, flow simulations was performed for a number of percolation models, and the possibility of using this method was investigated. By analyzing the simulation results, the following results are obtained:

1- For simulation cases with the either very low or very high occupancy probability (i.e., not close to the threshold), the breakthrough in the models with k≠0 for the non-reservoir parts of the reservoir is much faster (i.e., less time needed). This means that in such cases, the injected fluid will be able to communicate through other finite clusters (not necessarily the main clusters).

2. The impact of permeability contrast between reservoir and non-reservoir parts in the model is more pronounced when the occupancy is below the threshold. In such cases, the injected fluid faces more constraints to move through the reservoir and to direct towards the non-reservoir parts. When the occupancy is high, the fluid mainly moves through the high permeability regions.

3. In reservoirs with not significant permeability contrast between the reservoir and non-reservoir parts, the percolation method should be used with caution. In these cases, the flow is likely to pass through the finite clusters in addition to the spanning cluster.

4. The average breakthrough time increases with an increase in the occupancy and decreasing with the reservoir heterogeneity. Also, there is positive correlation between the injection volume needed for the breakthrough and the reservoir pore volume.

5. The number of realizations to get a reasonable statistics of breakthrough time increases in the case of heterogeneous models close to the threshold. Therefore, the number of realizations depend on the occupancy probability and reach a maximum near the threshold (p = 0.668) and then decreases with the occupancy.

6. The comparison of the breakthrough time of models with zero permeability background ($t_{k=0}$) and the breakthrough time of models with non-zero permeability background ($t_{k=\alpha k}$) with α=0.001 and k as permeability of reservoir part shows a positive correlation, which it can be used in developing the breakthrough time distribution of such cases. However, the detail dependency of these two times needs further study by using different α values to investigate the exact role of this α.

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