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A Cost-free Alternative Approach to Simulation of Pressure Transient Response for Slightly Compressible Fluids

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Abstract
Generating pressure transient response for an interpretation model to describe essential features of a reservoir system accurately is often difficult. It is generally due to the inaccessibility of standard pressure transient analysis tools due to the cost, and even when accessible, they are constrained to its workflow and limitations. This study presents an alternative to standard industry tools to determine transient pressure response for a given rate history. A reservoir model for a single well with constant skin and wellbore storage producing a varying step rate in a semi-infinite acting reservoir with a sealing fault was used as a case study. The well is also assumed to be producing above saturation pressure from a reservoir saturated with slightly compressible fluid hence having constant fluid properties. The method discussed in this study can be applied to well-test interpretation models with an analytical constant terminal rate solution producing at variable step rates from a reservoir having constant rock and fluid properties. The results show conformance with that of standard industry software, and diagnostic plots of the simulated data set can help engineers plan well-test jobs and study the behavior of different reservoir models. Moreover, the program can be modified and used to regress observed pressure response with a selected model. The approach suggested by this study is a perfect alternative where time and cost are constraints.

Keywords: Reservoir Model, Infinite-acting, Wellbore Storage, Skin, Physical Systems.

Introduction
Reservoir engineers and researchers are usually concerned with understanding the behavior of physical systems. They usually generate models to help them describe the quantitative and logical behavior of a physical system. These models may be physical (scaled-down representations of the original system) or mathematical models (empirical and physical laws expressed as systems of equations). Mathematical models are preferred where physical models are either infeasible or too expensive to build [1]. Mathematical models may be deterministic or probabilistic (stochastic) models. Probabilistic models consider the effect of random phenomena, while deterministic models ignore them.

Most often, engineers are more concerned with modeling special types of physical systems called dynamic systems. A dynamic system's output (dynamic) variables depend on their initial and previous values [1]. Deterministic models for the dynamic system are usually written as a system of differential equations. These equations may or may not have an analytical or exact solution, and the exact solutions are characteristic of linear systems of differential equations. Although numerical solutions may be obtained for mathematical models having an analytical solution, they are usually generated for mathematical models with no closed-form solution due to non-linearity and complex geometries [3].

The reservoir is a crucial part of the hydrocarbon asset, and it can be considered a dynamic system. In the oil industry, management makes decisions dependent on the understanding of the entire hydrocarbon asset. This requires the generation of integrated flow models of the hydrocarbon asset by a team of specialists, including geologists, geophysicists, petrophysicists, reservoir engineers, drilling/completion engineers, production engineers, petroleum economists etc. [4].

Well test interpretation models are critical to understanding the diffusion process occurring in the reservoir. These interpretation models are used to estimate reservoir system parameters (such as skin, permeability, etc.) used in building material balance models and evaluating the effectiveness of a stimulation job [3].

Usually, a reservoir engineer may wish to generate the pressure response for an interpretation model for two main reasons.

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The first reason is that in a well-test interpretation methodology, an interpretation model's generated pressure response is history matched against the pressure and rate data. Several pressures and rate data plots are made to determine if the selected model is adequate. Secondly, generating pressure response for an interpretation model can help answer "what if" questions that might be infeasible or expensive to answer experimentally. Finally, knowing how to generate pressure transient response for a reservoir model does not limit one to the methodology used by any given software, mainly aiding research [1-6].

This study focuses on using Julia language to generate transient pressure responses for slightly compressible fluid interpretation models with closed-form solutions. It uses the case of the interpretation model for a single well with constant (rate in-dependent) skin and wellbore storage producing at varying rates in a semi-infinite reservoir with a sealing fault. The reservoir is assumed to be produced above saturation pressure and saturated with a single-phase slightly compressible fluid; hence it has constant PVT parameters.

Materials and Methods

The methodology introduced in this study can be applied to generate responses for any interpretation model with an analytical solution for reservoirs having constant rock and fluid properties (i.e. slightly compressible oils) using 'Julia' provided it has uniform initial reservoir pressure. Julia is a high-level, high-performance dynamic programming language. It provides a cost-free alternative for the generation of pressure transient response alongside developer speed, readable code and the ability to optimize code for run-time speed.

The general methodology for determining the transient pressure response for well test interpretation models having analytical solutions is as follows:

1. Select a slightly compressible oil interpretation model with a dimensionless constant terminal rate analytical solution (neglecting skin and wellbore storage).
2. Create a Lookup table for the dimensionless pressure solution of the selected model using the Stephest algorithm.
3. Input rate-time history and create a step rate history generating function.
4. Add the effects of variable surface flow rate, skin and varying wellbore storage.

Step 1: Select a slightly compressible oil interpretation model with a dimensionless constant terminal rate analytical solution (neglecting skin and wellbore storage).

For the case study, a well with constant skin and wellbore storage producing slightly compressible fluid at a varying rate in a semi-infinite acting reservoir with a sealing fault, the model in SI units are given as:

$$\frac{\partial^2 P}{\partial x^2}+\frac{\partial^2 P}{\partial y^2}+\frac{\partial^2 P}{\partial z^2}=\frac{\varphi\mu c_t}{k}\frac{\partial P}{\partial t} \tag{1}$$

Initial condition: $P(x,y,z,0)=P_i \ \forall x,y,z$ (2)

Inner boundary condition:

$$\frac{q\mu}{2\pi kh}=\frac{x^2+y^2}{x}\frac{\partial P}{\partial x}\forall x,y,z \text{ such that } x^2+y^2=r_w^{\ 2} \tag{3}$$

Outer boundary condition: $\frac{\partial P}{\partial x}0 \text{ at } x=L_f \ \forall y,z$ (4)

where,

P = pressure at any position (x, y, z) and time t.
P_i = the initial reservoir pressure.
q = constant well production rate.
r_w = wellbore radius.
x,y,z = Cartesian distances from the origin.
L_f = perpendicular distance to the fault, which is taken along the x-axis.
k = reservoir effective permeability. ($k_x=k_y=k_z=k$)
μ = oil viscosity.
c_t = total compressibility.
h = reservoir thickness.
φ = reservoir porosity.
$(\varphi\mu c_t)/k$ = constant, making the partial differential equation a linear one.

The solution to this model is obtained by remodeling the problem using a technique known as the method of images (which considers a well equidistant from the boundary in an infinite reservoir). It is solved using the principle of superposition for multi-well infinite systems. Applying the principle of superposition to the multi-well problem, the dimensionless pressure Laplace space solution can be given as:

$$\widetilde{P_D}(1,p)=\frac{K_o(\sqrt{p})}{p^{\frac{3}{2}}K_1(\sqrt{p})}+\frac{K_o(L_{D2}\sqrt{p})}{p^{\frac{3}{2}}K_1(\sqrt{p})} \tag{5}$$

where,

L_D = dimensionless distance to boundary
p = Laplace variable
K_o (z) = modified Bessel's function of the second kind and zero-order.
K_1 (z)=-K_o' (z) = modified Bessel's function of the second kind and the first order.
K_o (z), K_1 (z) are computed using the "SpecialFunctions.jl" Julia module.

Furthermore, single sealing fault with corresponding image well after kappa is shown in Figure 1.

Step 2: Create a Lookup table for the time domain dimensionless pressure function of the selected model using Stephest algorithm.

After obtaining the analytical solution for the selected model, the next step involves generating a lookup table for the dimensionless pressure solution in the time domain. Loo essential features kup table is generated because the model's real-time analytical solution can be computationally tedious to compute.

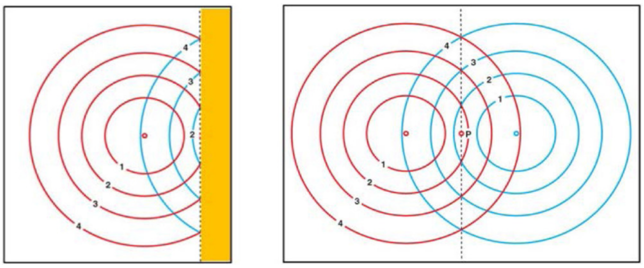

Fig. 1 Single sealing fault with corresponding image well (after kappa).

Analytical solutions may either be explicit real-time solutions, Laplace space solution or green function solution (pressure derivative) [2]. Laplace space solutions and green function solutions are considered computationally tedious, and lookup tables for these solutions are preferred. It is important to note

that explicit real-time solutions do not need a lookup table implementation as they are not computationally tedious.
For the case study, the Laplace space solution was inverted using the Stephest algorithm, and a lookup table was created using the inversion results for the range $2 \leq LD \leq 1e6$ and $9e-5 \leq TD \leq 2e18$ with 51 values equally spaced in log10 (LD) space and 101 equally spaced in log10 (Td) space [8]. The value of PD at a selected TD and LD is gotten from the lookup table by interpolating atlog10 (T_d) and log10 (L_d) Using B-Splines interpolation of degree 2, with the assumption of zero derivatives applied to the boundary grid points [7]. The B-spline interpolation was done using the "Interpolations.jl"' module in Julia.

Step 3: Input rate-time history and generate a step rate history generating function

Here, the rate-time history array was inputted. A higher-order function (function accepting functions as input or returning a function as output) is programmed to return a step-rate function from the inputted rate-time history. The returned rate function produces the value of the rate at any inputted time.
The step-rate function can be represented mathematically as:

$$q_s(T)=q_{s,N+1}+\sum_{i=1}^{N}(q_{s,i}-q_{s,i+1})*u(T-T_i) \quad (6)$$

$$u(T-T_i)=\begin{cases}1, \text{when } T\leq T_i\\ 0, \text{when } T>T_i\end{cases} \quad (7)$$

In vector form q(*t*) can be expressed as:

$$q_s(T)=q_{s,N+1}+\begin{pmatrix} q_{s,1}-q_{s,2}\\ q_{s,2}-q_{s,3}\\ \vdots\\ q_{s,i}-q_{s,i+1}\\ \vdots\\ q_{s,M}-q_{s,N+1}\end{pmatrix}^T \begin{pmatrix} u(T-T_1)\\ u(T-T_2)\\ \vdots\\ u(T-T_i)\\ \vdots\\ u(T-T_N)\end{pmatrix} \quad (8)$$

where,
q_s (T) = measured flow rate at time T.
$u(T-T_i)$ = unit step function
$q_{s,i}$ = measured rate at time T_i from the rate-time history.
$q_{s,i+1}$ = measured rate at time T from the rate-time history.
$q_{s,N+1}$= measured rate at time T_{M+1} from the rate-time history
q(s,N) = measured rate at time T_M from the rate-time history
N+1 = number of rate changes.
V^T = Transpose of a vector V

Step 4: Add the effects of variable surface flow rate, skin and (in general) varying wellbore storage

Here, the effects of variable flow rate, skin and wellbore storage were added to simulate the pressure response. For our case study, the effects of rate-independent skin, ideal (constant) wellbore storage and variable flow rate can be added using the discretized versions of the convolution equation, wellbore model and near wellbore model [2]. The discretized equations, when solved, yield the following equations, which simulate the transient pressure response.

$$a12=P_D((T_{M-T-}T_{M-1})\beta) \quad (9)$$

$$a_{22}=\frac{T_M-T_{M-1}}{C_s} \quad (10)$$

$$a_{32}=-\alpha S \quad (11)$$

$$c_1=P_o-\alpha\sum_{i=1}^{M-1}(q_i-q_{i-1})P_D((T_M-T_{i-1})\beta)+q_{M-1}P_D((T_M-T_{M-1})\beta) \quad (12)$$

$$c_2=a_{22}q_s(T_M)B-P_{wf}(T_{M-1}) \quad (13)$$

$$q(T_M)=\frac{c_1+c_2}{a_{12}+a_{22}-a_{32}} \quad (14)$$

$$P_w(T_M)=c_1-a\,q(T_M) \quad (15)$$

$$Pwf(T_M)=a_{22}q(T_M)-c_2 \quad (16)$$

where,
T_M = time at the current time-step M
T_{M-1}= time at previous time-step (M-1)
T_{i-1}= time at time-step (i-1)
$P_D(T_M*\beta)$ = dimensionless pressure at time T_M
$q(T_M)$ = sand face flow rate at time T_M
$P_w(T_M)$ = sand-face pressure at time step T_M
$P_{wf}(T_M)$ = wellbore pressure at time step T_M
q_{i-1} = sand-face rate at the (i-1)th time step
q_{M-1}= sand-face rate at the previous time step (M-1)
q_M= sand-face rate at the current time step,M.
$P_o=P_w(T_0)$ = initial reservoir pressure.
$q_s(T_M)$ = measured surface flow rate function at time, T_M
C_s = ideal wellbore storage coefficient.
B = Formation volume factor.
α and β are multipliers depending on the system of units used, rock and fluid properties.

The simulation was initiated using the model parameters shown in Table 1 with q_o=0, *M*=1 and α and *β* calculated from

$$\alpha=\frac{141.2\mu}{kh} \quad (17)$$

$$\beta=\frac{0.000264k}{\varphi\mu c_t r_w^2} \quad (18)$$

Table 1 Model parameters

Parameter (Unit)	Value
Initial Pressure, P_i (psia)	5000
Distance to sealing fault, L (ft)	1000
Pay zone, h (ft)	100
Well radius, r_w (ft)	0.5
Formation compressibility, c_t (psi^{-1})	1.5 E – 6
Wellbore storage coefficient, C (bbl/psi)	0.01
Skin, s	0
Porosity, (-)(%)	20
Permeability, K (md)	100
Viscosity, µ (cp)	1.2
Formation volume factor, B (bbl/STB)	1.2

A variation of the one-step, two-step adaptive time step control was used in simulating the pressure response in an attempt to reduce the computational effort. The one-step two-step time controls were embedded into the program computing the pressure response [5].
The time control was based only on feedback from the measured wellbore pressure $P_{wf}(T_M)$. A tolerance of 0.1 psia was considered sufficient for the simulation. The local error estimate is also given by:

$$e_{m+1}=P_{wf,m+1}*(h/2)-P_{wf,m+1}*(h) \quad (19)$$

and

$$q=\left|\frac{e_{m+1}}{tol}\right| \quad (20)$$

The step size irrespective of the value of q is given by:

$$h_{m+1}=0.95*h_m*\min(\max\sqrt{((1/q),0.5)},2) \quad (21)$$

The 0.95 is a safety factor to help reduce chances that a step gets rejected, while 2 and 0.5 control the maximum and min imum factor, respectively, by which h changes in a single step.

A h_{min} and h_{max} time-step was also specified where,

If $h_{m+1}<h_{min}$ (22)

$\rightarrow h_{m+1}=h_{min}$ (23)

Else if $h_{m+1}>h_{max}$ (24)

$\rightarrow h_{m+1}=h_{max}$ (25)

If q<1, the step is accepted, and the refined solution at T_{m+1} becomes

$P_{wf,m+1}=P_{wf,m+1}$* (h/2)+ $e_{m+1}/3$ (26)

The equations for dealing with rate-dependent skin and non-ideal storage are discussed by Stewart [2]. The results of the simulated pressure response were compared with those obtained from standard industry software in the results section.

Results and Discussion

In Figure 2, the simulated pressure transient response obtained from Julia is shown. As seen from the figure, the simulated response produces a smooth response. For each flow period (period of constant rate production), the pressure drop (relative to the initial pressure) increases rapidly initially and stabilizes at a late time. Flow periods with higher production rates have higher pressure drops (relative to the initial pressure) than flow periods with lower production rates.

In Figure 3, the simulated pressure response obtained from industry software is shown. Figure 2 shows conformance with Figure 3, indicating that the pressure response obtained from Julia is similar to that obtained from industry software. This can be seen in the match in Figure 4.

Using Julia "BenchmarkTools.jl" module, the simulation run time was about 90 minutes, although a faster run time speed may be achieved if better adaptive time-step control algorithms are deployed.

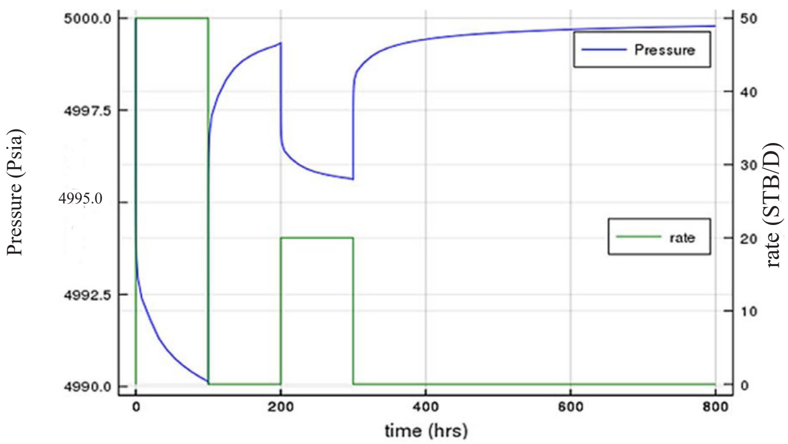

Fig. 2 Simulated pressure response using Julia.

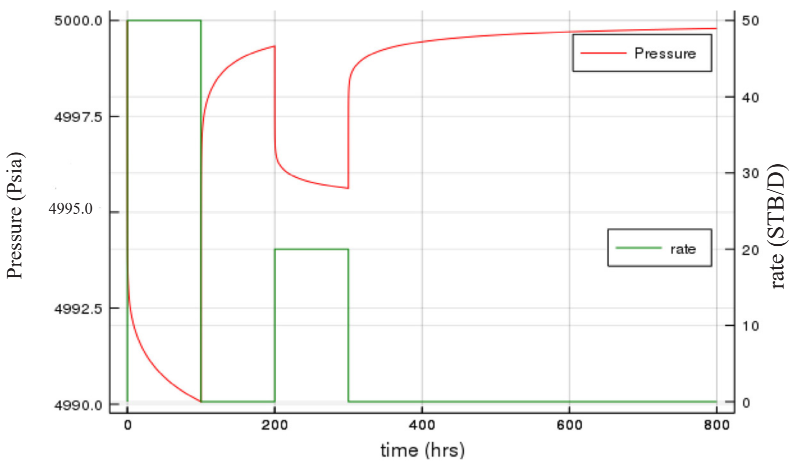

Fig. 3 Simulated pressure response using Industry software.

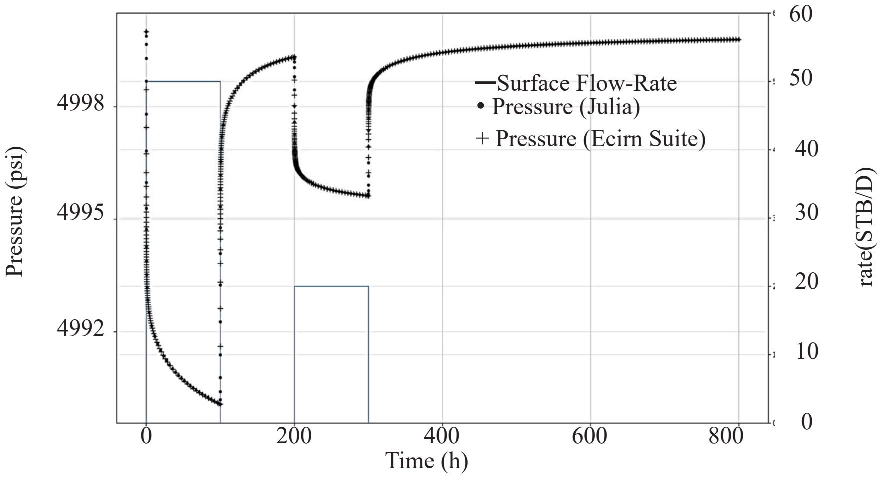

Fig. 4 Overlay of Simulated pressures from Industry Software and Julia.

Conclusions

Modeling and generating a response of dynamic systems is essential as it enables reservoir engineers to understand the behavior of the dynamic system adequately, and it helps in model matching, designing of well-tests and for research purposes.

In this study, a methodology for the simulation of pressure response for slightly compressible fluid interpretation models has been adequately discussed.

The results indicate that the generated pressure response behaves as expected and conforms to results obtained from typical industry software. This technique can be used as a quick and cheap pressure transient response analysis tool where time and cost are constraints.

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