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Numerical Modeling of the Amount and Rate of Sand Produced in Oil Wells

Ali Zalakinezhad and Saeid Jamshidi*
Department of Chemical and Petroleum Engineering, Sharif University of Technology, Tehran, Iran

Abstract
Nowdays, sand production is one of the most important challenges in the oil and gas industries, making numerous issues. To prevent these problems, it is necessary to use mathematical models to estimate the sand production onset and the amount of sand produced during production. There are generally four methods for predicting sand production: experimental methods that use field observations and well data, laboratory simulations, numerical methods, and analytical methods. In this research, a novel numerical method is proposed to estimate the amount of sand production. First, it is necessary to estimate the onset of sand production using failure criteria and after that, the amount of sand production is estimated. First, to use numerical methods, they must be calibrated by using field data. In this paper, the proposed numerical model is calibrated by using the field observations and well data of a North Sea reservoir. It is used to predict the amount of produced sand that the average relative error of the proposed method was about 6.9%. Also, in this model, computable parameters are used to calculate the amount of sand production, which reduces the error of this method. It also shows that this is a practical model. Therefore, the proposed model is reliable, and it can be used to estimate the amount of sand production for subsequent years. The proposed model is developed based on incompressible and slightly compressible fluids; this paper also considers the relationship between porosity and permeability at steady-state conditions. Ultimately, sensitivity analysis on sand production is performed, and the effects of four permeability parameters: uniaxial compressive strength, maximum horizontal stress, and wellbore pressure on sand production are checked.

Keywords: Sand Production, Numerical Modeling, Oil and Gas Wells, Failure Criteria.

Introduction
The production of formation sands and formation fluids (oil, gas, and water) is called sand production [1]. Due to the interaction between the fluid and the formation particles, the grains and particles can separate from the formation fluid and enter the well.
A significant portion of the world's oil and gas reserves are in weak sandstone reservoirs and prone to sand production [2]. Almost 70% of the world's hydrocarbons are in unconsolidated reservoirs, increasing the importance of paying attention to the management and forecasting of sand production [3]. Poor adhesion between grains in weak sandstone reservoirs is one of the main factors for sand production in these reservoirs [2].
In Iran, sand production has been reported in some fields, among which, the production of sand in Cheshmeh Khosh field, which is an oil field, and Sarkhoon and Gonbadli fields, which are gas fields can be mentioned by us. Also, there is sand production in some wells of Ahvaz Mansouri field and Darkhovin field. Recently, the problem of sand production has also been studied in the fields of the Caspian Sea basin [4].
Sand production has created many problems for the oil and gas industries. Ultimately, sensitivity analysis on sand production is performed, and the effects of four permeability parameters: uniaxial compressive strength, maximum horizontal stress, and wellbore pressure on sand production are checked. These problems are as follows [5-8]:
- Reducing production rates due to damage to production path;
- Well instability and well closure;
- Equipment damage and equipment erosion;
- Loss of time due to repair of damaged equipment due to sand production;
- Loss of well control;
- Separating sand grains from oil is a time-consuming and costly operation;
- The fluid's viscosity and density increase when it mixes with the sand, so fluid lifting costs and energy will increase.

**Corresponding author:* Saeid Jamshidi, Department of Chemical and Petroleum Engineering, Sharif University of Technology, Tehran, Iran
E-mail addresses: jamshidi@sharif.edu
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As mentioned, one of the problems of sand production is equipment erosion. One of the applications of modeling sand production is to predict equipment erosion. There are many equations based on equipment geometry and size, size, density and amount of produced sand and fluid properties, and some other parameters that can be used to predict equipment erosion [9].
To prevent the above problems, it is necessary to model sand production to make a more efficient production system design to avoid excessive sand production problems.
In the following, the mechanisms of sand production, methods of controlling sand production, and methods of predicting the amount of sand production are described.

Mechanisms of Sand Production

To model sand production, two mechanisms are required to be combined. The first mechanism is the mechanical instability around the wellbore. The second mechanism is the hydrodynamic instability due to the pressure gradient caused by the fluid flow on the degraded material around the wellbore.
Mechanical instability around the well can be divided into shear failure and tensile failure. During production and when the bottomhole pressure is low, shear failure is possible, and tensile failure is possible during injection and when the bottomhole pressure is high. The mechanical instability can be estimated by using failure criteria.
When mechanical instability occurs around a well, as soon as fluid production begins, hydrodynamic instability arises, and sand is produced along with the fluid.

Sand Control Methods

There are different methods for controlling sand production: mechanical sand control, chemical sand control, sand control during production, and the management of produced sand on the surface. It should be noted that the choice of the proper methods for controlling sand production does not follow a specific law, and there are no relationships between them [10, 11].
In the sand control during production, specifying the critical production rate and the critical pressure, the pressure and the production rate are not allowed to pass the critical values. In managing produced sand on the surface, sand production is allowed, and then by using some equipment, sand is separated from fluids in the surface [11].
In chemical methods, chemicals such as resin are injected into the well for controlling sand. The most widely used methods of sand control are mechanical. Among these methods, the use of gravel packs to prevent sand production can be found in our previous work [12].

Critical Drawdown Chart

As mentioned earlier, one of the sand control methods is to determine critical wellbore pressure, and it is not allowed the wellbore pressure to pass the critical pressure.
In the critical drawdown chart, critical wellbore pressure plot versus reservoir pressure, and thereby, critical drawdown pressure is obvious every time. An example of a critical drawdown chart is shown in Figure 1.

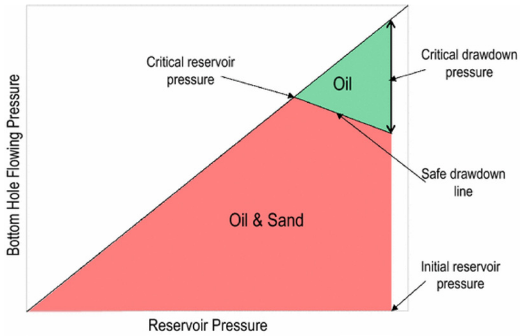

Fig. 1 An example of critical drawdown pressure chart [13].

The onset of sand production and the critical wellbore pressure can be calculated according to failure criteria. The appendix shows the equations for calculating critical wellbore pressure according to the Mohr-coulomb failure criterion and Mogi-coulomb failure criterion. When wellbore pressure becomes less than critical wellbore pressure, sand production will be started.

Methods for Predicting Sand Production

There are generally four methods for predicting sand production: experimental methods that use field observations, laboratory simulations, numerical methods, and analytical methods. Numerical models provide a complete description of the stress state and give a better and more accurate answer than other methods. They consider more rock failure and sand production factors if all required input data are available. The most important drawback of this method is its complexity and time consumption [14].
In the experimental method, an attempt is made to obtain a correlation between sand production and reservoir parameters and production and completion parameters. The more parameters are used, the more accurate the method is, and the resulting correlation may not be very accurate for all repositories [15].
In the laboratory simulation method, sand production is simulated using samples obtained from the reservoir. The main problem with this method is that it requires many reservoir samples, so it is a costly and time-consuming method [16].
Analytical methods are speedy and easy to use. This method's prediction accuracy depends on the selected failure criterion, model the rock's behavior, and the accurate measurement of parameters affecting the rock failure. Analytical methods have limitations that do not have valid results on a field scale [2, 16].
Numerical methods will probably not be possible if there is not enough time, data, and information. Of course, hypothetical or approximate data can be used instead of non-existent data, but the resulting answer is probably less accurate than other methods [16].
Given the importance of predicting sand production, considerable efforts have been made to develop numerical methods for predicting sand production. Numerical methods are the strongest and most efficient tools for predicting sand production, and they can also be combined with analytical methods. Laboratory results, field observations, and well data are used to calibrate numerical models [2].
The proposed model is developed based on incompressible

fluids (also, it can be used for slightly compressible fluids). At steady-state conditions, the relation between porosity and permeability is based on Kozeny–Carman relation. The type of fluid and the relation between porosity and permeability are not considered in related studies. Also, in this study, two different failure criteria are used to estimate the onset of sand production. In related articles, estimation of the onset of sand production is less considered.
In this study, the proposed numerical model is calibrated and used to estimate the amount of produced sand by using actual well and field data. The results show that the proposed model has reliable results.

Theory

In the last three decades, many numerical methods have been proposed. Some of the most important of them are introduced below.
In 1996, it was proposed by Vardoulakis et al that the following equation for calculating the amount of sand produced, in which only the hydromechanical effects are considered in this equation and mechanical instability is not considered [17, 18].

$$\rho_s \times \frac{d\varnothing}{dt} = \frac{dm}{dt} = (1-\varnothing)\times\lambda\times c\times q_i \tag{1}$$

In the above equations, ρ_s is the density of sand grains, $\varnothing$ indicates porosity, t is time, m is the amount of sand produced, λ indicates the sand production coefficient, c shows the concentration of solids that dissolved in the fluid, c_{cr} indicates the critical concentration of solids that dissolved in the fluid, ‖ ‖ indicates the Euclidean norm function, and q_i indicates the amount of production.
Skjaerstein et al. in 1997 considered the following equation to estimate the amount of sand produced, which is almost identical to the above equations. In this equation, there is no trace of critical wellbore pressure, and consequently, the effects of mechanical instability [17].

$$\rho_s \times \frac{d\varnothing}{dt} = \frac{dm}{dt} = \lambda\times c\times\left(\varnothing-\frac{\varnothing^2}{\varnothing_{cr}}\right)\times q_x \tag{2}$$

In the above equation, $\varnothing_{cr}$ indicates critical porosity, and q_x is the amount of production flux.
In 1998, Papamichos and Stavropoulou modified the relationship proposed by Vardoulakis et al. in 1996 as follows and related sand production coefficient to mechanical instability and plastic strain [19]:

$$\rho_s \times \frac{d\varnothing}{dt} = \frac{dm}{dt} = (1-\varnothing)\times\lambda(\varepsilon_p)\times c\times q_i \tag{3}$$

$$\lambda(\varepsilon_p) = \begin{Bmatrix} 0 & if\ (\varepsilon_p < \varepsilon_{p_{cr}}) \\ \lambda_1\left(\varepsilon_p - \varepsilon_{p_{cr}}\right) & if (\varepsilon_p > \varepsilon_{p_{cr}}) \\ \lambda_2 & if\ \lambda_1\left(\varepsilon_p - \varepsilon_{p_{cr}}\right) > \lambda_2 \end{Bmatrix} \tag{4}$$

In the above equation, ε_p represents the amount of plastic strain, $\varepsilon_{p_{cr}}$ represents the critical plastic strain when mechanical instability occurs, and λ1 and λ2 are the sand production coefficients used to calibrate the relationship by field data or laboratory data.
Papamichos and Malmanger, in 1999, used the following equation to calculate the amount of sand produced, in which both mechanical instability and hydromechanical instability were considered, and the Mohr-Coulomb failure criterion was used [20].

$$\rho_s \times \frac{d\varnothing}{dt} = \frac{dm}{dt} = (1-\varnothing)\times\lambda(\varepsilon_p)\times c\times\sqrt{q_i q_i} \tag{5}$$

In 2001, Weng and Xue proposed a new relationship for the sand production coefficient relationship that was introduced in 1998 by Papamichos and Stavropoulou, which is as follows [21]:

$$\lambda(\varepsilon_p) = \begin{Bmatrix} 0 & if\ (\varepsilon_p < \varepsilon_{p_{cr}}) \\ \lambda_1\left(\varepsilon_p - \varepsilon_{p_{cr}}\right) & if\ (\varepsilon_{p_{cr}} + \frac{\lambda_2}{\lambda_1} > \varepsilon_p > \varepsilon_{p_{cr}}) \\ \lambda_2 & if\ (\varepsilon_{p_{cr}} + \frac{\lambda_2}{\lambda_1} < \varepsilon_p) \end{Bmatrix} \tag{6}$$

Another numerical relationship that is used in forecasting sand production is as follows, which is used in the current research [22]. The following numerical model was chosen because it includes computable parameters such as porosity, production flux, reservoir fluid viscosity, and sand grain density.
Relationships based on numerical models have many parameters, which can lead to the use of hypothetical values for some parameters. And as an outcome, the resulting answer is associated with a large error. Therefore, using a numerical model that requires fewer parameters can be more useful.

$$\frac{\partial\varnothing}{\partial t} = \frac{\lambda_{sand}\times\mu_f}{\rho_s}\times\frac{1-\varnothing}{\varnothing^3}\times\left(q_{fl} - q_{fl}^{\ cr}\right) \tag{7}$$

In the above equation, µf denotes the viscosity of the reservoir fluid in centipoise, q_{fl} is the rate of production flux in m/s, and $q_{fl}^{\ cl}$ is equal to the critical production flux in m/s. The critical production rate is based on the critical pressure drop, and the critical pressure drop can be calculated by using failure criteria, and λ_{sand} represents the sand production coefficient in s/m^3.

Sand Production Prediction

One of the methods that can be used to estimate the amount of sand produced is to use the change of porosity around the wellbore due to sand production. In the area around the well, due to the production of sand, the porous space increases, and as a result, the porosity increases. In the below equation, the relationship between the changes in porosity and the amount of sand produced can be seen [22].

$$\frac{\partial m_{sand}}{\partial t} = \rho_s\times\frac{\partial\varnothing}{\partial t}\times V_{sp} \tag{8}$$

In Equation 8, V_{sp} indicates the volume around the well, which contributes to sand production, and m_{sand} indicates the total volume of sand produced, which is in kilograms.
The first step in predicting sand production is determining the type of formation. The first data required to estimate the amount of sand produced is the critical wellbore pressure that can be calculated using failure criteria. Any prediction of sand production starts by estimating the onset of sand production. It can be said that formation strength, production rate, and in situ stresses are the most important data needed to assess rock failure time [16, 23].
To calculate the amount of sand production, it is necessary to calculate the porosity change around the wellbore. Note that during production, the porosity of almost all the reservoir is constant, and only the porosity of a small volume around the wellbore will change. Note that the sand production is from the mentioned volume, and only a small volume around the

wellbore is involved in sand production [22].

$$\frac{\partial \varnothing}{\partial t}=\frac{\lambda_{sand}\times\mu_f}{\rho_s}\times\frac{1-\varnothing}{\varnothing^3}\times\left(q_{fl}-q_{fl}^{cr}\right) \quad (9)$$

There is no equation to calculate the value of the sand production coefficient parameter, and this parameter must be obtained based on the actual data of the wells. In the study which was carried out by Son Thung, by using a series of data, an optimal value for this parameter which is equal to 0.0016667 s/m^3 was obtained [24].

To numerically solve Equation 8, first, it is necessary to discretize that like below.

$$\varnothing_{t_2}-\varnothing_{t_1}=\frac{\lambda_{sand}\times\mu_f}{\rho_s}\times\frac{1-\varnothing_{t_1}}{\varnothing_{t_1}^{\ 3}}\times\left(q_{fl}-q_{fl}^{cr}\right)\times\Delta t \quad (10)$$

The porosity at the current time can be calculated by Equation 10 by using the required values at the previous time.

$$\varnothing_{t2}=\varnothing_{t1}+\frac{\lambda_{sand}\times\mu_f}{\rho_s}\times\frac{1-\varnothing_{t1}}{\varnothing_{t1}^{\ 3}}\times\left(q_{fl}-q_{fl}^{cr}\right)\times\Delta t \quad (11)$$

To solve Equation 11, first, it is necessary to calculate the value of produced flux. According to Darcy's law, the rate of production is calculated from Equation 11.

$$q=\frac{7.08\times k\times A}{\mu}\times\frac{\partial p}{\partial r} \quad (12)$$

In Equation 12, k represents permeability in Darcy, p represents the fluid pressure in terms of psi, r indicates the radius in terms of feet, A means the area in terms of square feet, and the rate of production is obtained in bbl/day.

It is assumed that reservoir geometry is radial, so it is possible to take an integral of Equation 12 as below. Note that it is assumed that flow rate, permeability, and viscosity are constant.

$$\int_{r_w}^{r_e}\frac{1}{2\pi rh}\partial r=\int_{p_w}^{p_r}\frac{1.127\times k}{\mu\times q}\times\partial p \quad (13)$$

By solving the above integral, the following equation can be obtained. Notice that as production flux is necessary, therefore, the result must be divided by (A=2πrh).

$$q=\frac{7.08\times k\left(D\right)\times\Delta p\left(psi\right)}{2\pi\times h\left(ft\right)\times\ln\left(\frac{r_e}{r_w}\right)\times\mu} \quad (14)$$

Since the unit of production flux required in the existing equations is meters per second, it is necessary to change the unit based on Equation (15).

$$q\left(\frac{m}{s}\right)=1.980847\times10^{-5}\times\frac{7.08\times k\left(D\right)\times\Delta p\left(psi\right)}{2\pi\times h\left(ft\right)\times\ln\left(\frac{r_e}{r_w}\right)\times\mu}=$$

$$2.232\times10^{-5}\times\frac{k\times\Delta p}{\mu\times h\times\ln\left(\frac{r_e}{r_w}\right)} \quad (15)$$

In Equation 15, Δ_p represents the pressure drop in psi, h represents the reservoir thickness in feet, r_w represents the well radius in feet, r_e represents the reservoir radius in feet, and q represents the production flux in m/s.

To calculate the critical production rate, it is necessary to put the critical pressure drop in Equation 15.

$$q^{cr}=2.232\times10^{-5}\times\frac{k\times\Delta p^{cr}}{\mu\times r_w\times\ln\left(\frac{r_e}{r_w}\right)} \quad (16)$$

To calculate the critical pressure drop, it is necessary to calculate the critical wellbore pressure. Critical wellbore pressure can be calculated according to the Mohr-Coulomb failure criterion or Mogi-coulomb failure criterion that the necessary equations are in appendixes.

$$\Delta p^{cr}=reservoir\ current\ pressure-p_{w_{cr}}\left(from\ mohr\ or\ mogi\right) \quad (17)$$

According to the given information, the difference between the production flux and the critical production flux can be calculated from Equation 18.

$$q_{fl}-q_{fl}^{cr}=2.232\times10^{-5}\times\frac{k}{\mu\times r_w\times\ln\left(\frac{r_e}{r_w}\right)}\times\left(\Delta p-\Delta p^{cr}\right) \quad (18)$$

In the first time, the porosity is equal to the initial porosity, and the permeability is equal to the initial permeability.

As the porosity changes, the permeability also changes. There are several equations for the relationship between porosity and permeability; in this article, Kozeny–Carman relation is used [25].

$$k_{t_i}=k_{t_1}\times\left(\frac{\varnothing_{t_i}^{\ 3}}{\left(1-\varnothing_{t_i}\right)^2}\right)\times\left(\frac{\left(1-\varnothing_{t_1}\right)^2}{\varnothing_{t_1}^{\ 3}}\right) \quad (19)$$

In Equation 19, k_{ti} represents the permeability at the current time, k_{t1} represents the initial permeability, and ti represents the porosity at the current time.

According to the relations mentioned so far, the general equation for calculating the porosity is obtained as follows:

$$\varnothing_{t_i}=\varnothing_{t_{i-1}}+\frac{\lambda_{sand}}{\rho_s}\times\frac{1-\varnothing_{t_{i-1}}}{\varnothing_{t_{i-1}}^{\ 3}}\times\left(2.232\times10^{-5}\times\frac{\kappa_{\tau_{i-1}}}{\rho_\omega\times\lambda v\left(\frac{\rho_\varepsilon}{\rho_\omega}\right)}\right)\times\left(\Delta p-\Delta p^{cr}\right)\times\Delta t \quad (20)$$

After calculating the porosity at any time, the amount of sand produced can be obtained based on Equation 8, which after discretization, Equation 8 is as follows:

$$m_{t_i}-m_{t_{i-1}}=\rho_s\times\left(\varnothing_{t_i}-\varnothing_{t_{i-1}}\right)\times V_{sp} \quad (21)$$

Therefore, the cumulative sand production at the current time can be calculated by:

$$m_{t_i}=m_{t_{i-1}}+\rho_s\times\left(\varnothing_{t_i}-\varnothing_{t_{i-1}}\right)\times V_{sp} \quad (22)$$

where, ρs represents the density of sand grains in kg/m^3, $\varnothing_{ti}$ indicates the porosity of the volume that is involved in sand production, at this time, $\varnothing_{ti-1}$ indicates the porosity of the volume that is involved in sand production, at the previous time, V_{sp} represents the amount of volume that is involved in the production of sand, the unit of which is cubic meters, m_{ti-1} represents the total amount of sand production until the previous time, the unit of which is kilograms and m_{ti} represents the total amount of sand produced until current time, the unit of which iskilograms.

The radius around the well, which is involved in the production of sand, is calculated by the below equation. Also, by Equation 21, the part of the reservoir involved in the production of sand can be calculated [22].

$$r_s=r_w\times e^{\left(\frac{\Delta p-\Delta p^{cr}}{C_0}\right)} \quad (23)$$

$$V_{sp}=\pi\times h\times r_w^{\ 2}\times\left(0.3048^3\right)\times\left(\left(e^{\left(\frac{\Delta p-\Delta p^{cr}}{C_0}\right)}\right)^2-1\right) \quad (24)$$

In the above equation, h represents the thickness of the reservoir in feet, rw represents the well radius in feet, Δp represents the pressure difference between the current reservoir pressure and the current wellbore pressure in psi, and Δp^{cr} represents the critical pressure difference that if the pressure drop is higher than Δp^{cr}, sand is produced, C_0 represents the uniaxial compressive strength in psi, V_{sp} indicates the volume involved in the production of sand in m^3. As mentioned earlier, the permeability in the sand production area will change. Still, the permeability in the rest of the reservoir will not change, so it is necessary to calculate the average permeability. There are different methods for calculating average permeability; this research uses the average harmonic method for radial geometry.

$$k_{avg} = \frac{\ln\left(\frac{r_e}{r_w}\right)}{\frac{\ln(\frac{r_e}{r_s})}{k_1} + \frac{\ln\left(\frac{r_s}{r_w}\right)}{k_s}} \tag{25}$$

In Equation 25, r_s is the radius of the area that is involved in sand production, k_s is the permeability of that area, and k_1 is the permeability of the rest of the reservoir.

After calculating the amount of sand produced, the rate of sand produced at the current time can also be obtained:

$$R_{sand_{t_i}} = \frac{m_{t_i} - m_{t_{i-1}}}{\Delta t} \tag{26}$$

In Equation 26, Δ_t is a time step in seconds, m_{t_i} and $m_{t_{i-1}}$), respectively, are the amount of sand produced at the current the previous time in kilograms and $R_{sand_{ti}}$ is equal to the rate of sand production at the current time in kg/s.

The process of obtaining the amount of sand produced is shown in Figure 2.

The required parameters to calculate the amount of sand produced is specified in Table 1.

Input data can be divided into four categories: geomechanical data, reservoir data, well data, and also sand data, which are shown in Table 2.

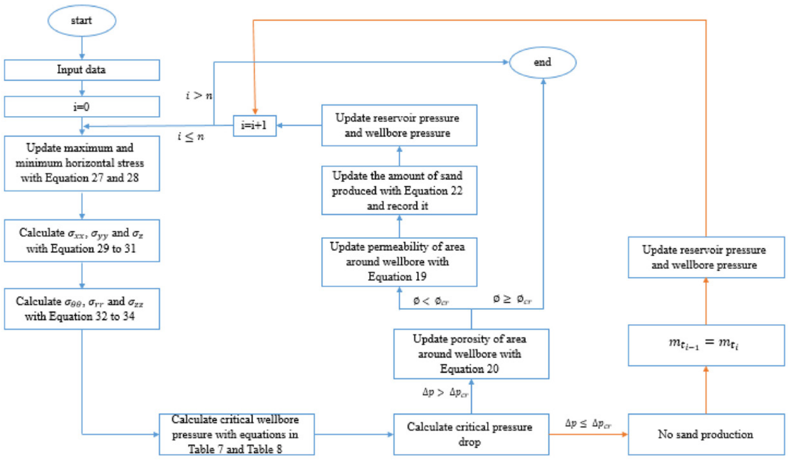

Fig. 2 The process of calculating the amount of sand production.

Table 1 The required parameters to calculate the amount of sand produced

parameter	unit
Vertical stress (σ_v)	psi
Maximum horizontal stress (σ_H)	psi
Minimum horizontal stress (σ_h)	psi
Inclination (i)	degree
Azimuth (α)	degree
Bottomhole pressure (p_w)	psi
Initial reservoir pressure (p_{ri})	psi
Current reservoir pressure (p_r)	psi
Poisson ratio (υ)	dimensionless
Biot coefficient (α)	dimensionless
Failure angle (β)	degree
Uniaxial compressive strength (C_0)	psi
Initial porosity (ϕ_i)	dimensionless
Critical porosity (ϕ_C)	dimensionless
Fluid viscosity (μ_f)	cP
Solid density (ρ_s)	kg/m^3
Well radius (r_w)	ft
Reservoir radius (r_e)	ft
Reservoir thickness (h)	ft
Reservoir permeability (k)	Darcy
Sand production coefficient (λ_{sand})	sec/m^3
Time step (Δt)	sec
Number of steps (n)	dimensionless
Failure model	-

Table 2 Different categories of input data.

Geomechanical data	Reservoir data	Well data	Sand data
Uniaxial compressive strength	Reservoir permeability	Inclination	Solid density
Failure angle	Reservoir porosity	Azimuth	Sand production coefficient
Biot coefficient	Reservoir thickness	Bottomhole pressure	-
Poisson ratio	Reservoir radius	Well radius	-
Vertical stress	Initial reservoir pressure	-	-
Maximum horizontal Stress	Current reservoir pressure	-	-
Minimum horizontal Stress	Fluid viscosity	-	-

At any time, based on the value of some parameters such as reservoir current pressure, bottomhole pressure, failure angle, uniaxial compressive strength, and other parameters, and with the help of one of the failure criteria, the critical wellbore pressure should be calculated. After calculating the value of critical bottomhole pressure and according to the value of current bottomhole pressure, one of the following two conditions occur:

• If the current wellbore pressure is greater than the critical wellbore pressure, in this case, the porosity at the current step time does not change compared to the porosity at the previous time. Consequently, the permeability and the amount of sand produced do not change.

• If the current wellbore pressure is less than the critical wellbore pressure; in this case, first, the amount of porosity in the new time step should be calculated by Equation 20, and based on that, the amount of permeability and then the amount of sand produced in the new time should be calculated by Equations 19 and 22. Then the rate of sand production in this time should be calculated by Equation 26.

Results and Discussion

In the following, by using the observed field data of a well and reservoir, the amount of sand produced calculated by the proposed method (Equation 22) is compared to observed field values. Then the effects of permeability, wellbore pressure, uniaxial compressive strength, and the maximum horizontal stress are investigated on the sand production.

The information and data related to a North Sea reservoir are shown in Table 3 [20]. Due to lack of information, some parameters such as uniaxial compressive strength, Biot coefficient, external radius, well radius, and failure angle have been assumed.

Table 3 Available data of the studied well and reservoir

parameter	unit	Value
σ_v	psi	6947.31
σ_H	psi	6483.19
σ_h	psi	6294.64
inclination	degree	50
Azimuth	degree	0
p_{r_i}	psi	5511.43
p_r	psi	4206.09
υ	dimensionless	0.20
α	dimensionless	1
β	degree	67.5
C_0	psi	3500
r_w	ft	2
r_e	ft	328.08
h	ft	390.09
k	Darcy	0.1
ϕ_i	dimensionless	0.22
ϕ_C	dimensionless	0.5
λ_{sand}	sec/m^3	0.00044
ρ_s	kg/m^3	2650
μ_f	cP	5
Δt	sec	3600
Failure Model	-	Mohr-coulomb

In Figure 3, the cumulative sand production and drawdown pressure of the mentioned well from the North Sea reservoir in 120 hours are shown.
As shown in Figure 3, based on wellbore pressure, the time interval can be divided into three different parts that each part has a unique wellbore pressure. The time and also wellbore pressure of each part are shown in Table 4.

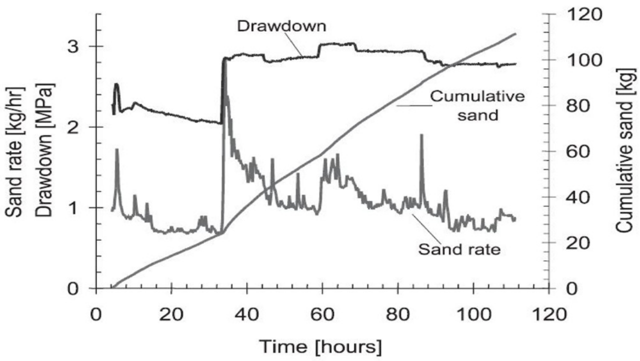

Fig. 3 Sand rate, cumulative sand production, and drawdown pressure for mentioned well during 120 hours period [20].

Table 4: Wellbore pressure in three different time intervals for the studied well.

Step	Time (hour)	Wellbore pressure (psi)
1	0-29	3887
2	30-84	3786
3	85-110	3800

To estimate the reliability of the proposed method, in this section, the results of the proposed method are compared with the field data. First, it is necessary to estimate the sand production coefficient.
The information of the sand production for the first part can be used to calibrate Equation 20 and obtain the value of the sand production coefficient, which is equal to 0.00044 s/m^3. To get the best value of the sand production coefficient, in different values of the sand production coefficient, the results of the proposed model for the first part are compared with the field results, and the best value, which is 0.00044 s/m^3, is selected. In Figure 4, the comparison between different values of sand production coefficient to select the best one is shown.

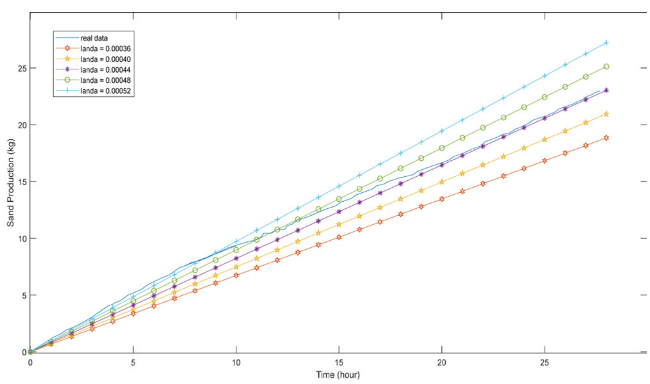

Fig. 4 Select the best sand production coefficient.

Now that the value of the sand production coefficient has been obtained, it is possible to simulate the amount of sand production by using Equation 22. A comparison of the actual cumulative sand production with the simulated cumulative sand production in the first interval is shown in Figure 5.

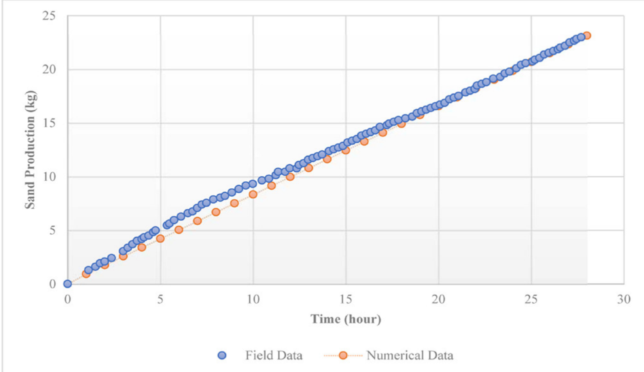

Fig. 5 Comparison of field data and simulation data for the amount of sand produced in the first interval.

To have more reliable answers, after each interval, it is necessary to update the porosity and permeability with Equations 20 and 19. In Table 5, the porosity and permeability at the start of each interval for the first interval are shown; in addition, porosity is equal to the initial porosity, and permeability is equal to the initial permeability.

Table 5 The porosity and permeability at the start of each interval.

	Porosity (fraction)	Permeability (darcy)
First interval	0.22	0.1
Second interval	0.2202	0.1003
Third interval	0.2205	0.1009

Now, the value of sand production coefficient has been obtained, the actual value and the simulated value of cumulative sand production in the second and third intervals can be compared to each other, which has been shown in Figures 6 and 7.

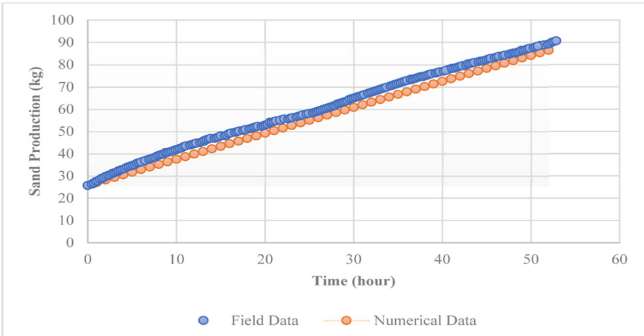

Fig. 6 Comparison of field data and simulation data for the amount of sand produced in the second interval.

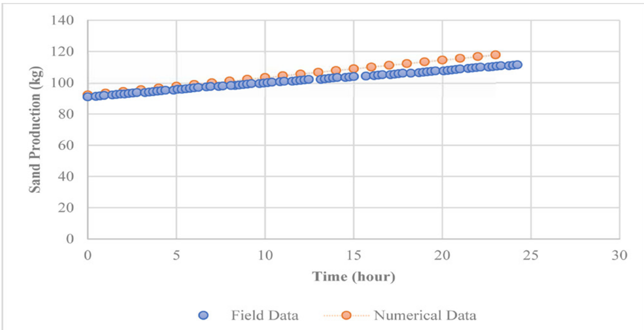

Fig. 7 Comparison of field data and simulation data for the amount of sand produced in the third interval.

According to Figures 5 to 7, the average relative error of the proposed method is around 6.9% which shows the reliability of the proposed method.

Sensitivity Analysis of Sand Production with proposed model

The hypothetical data in Table 6 were used to perform this sensitivity analysis. The below graphs are plotted by assuming that the reservoir pressure and wellbore pressure are constant during this period.

Table 6 Hypothetical data that are used to perform sensitivity analysis on sand production.

parameter	value	unit
σ_v	7000	psi
σ_H	4000	psi
σ_h	4000	psi
inclination	0	degree
Azimuth	0	degree
p_w	2000	psi
p_{r_i}	3000	psi
p_r	3000	psi
υ	0.25	dimensionless
α	0.7	dimensionless
β	55	degree
C_0	5000	psi
r_w	10	ft
r_e	600	ft
h	100	ft
ϕ_i	0.2	dimensionless
ϕ_C	0.5	dimensionless
λ_{sand}	0.1	sec/m^3
ρ_s	2650	kg/m^3
μ_f	2	cP
Δ_t	1000	sec
n	100	dimensionless
Failure Model	Mohr-coulomb	-

Permeability

According to Equation 20, permeability is directly related to the porosity, so, as the permeability increases, the porosity increases, and the amount of sand production increases. By using the data in Table 4 and considering four different values for permeability (0.01, 0.1, 0.2, and 0.3 Darcy), Figure 8 is obtained, which shows that when permeability increased, the amount of sand production also increases.

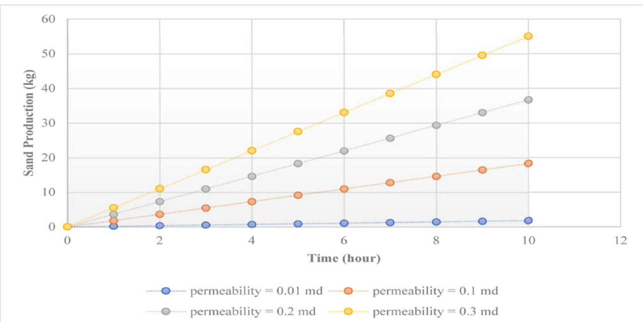

Fig. 8 Sensitivity analysis on accumulated sand production based on permeability parameter.

Uniaxial Compressive Strength

As the uniaxial compressive strength increases, the rock strength increases [26]; therefore, sand production decreases. The amount of sand production in three different values of uniaxial compressive strength (4800, 4900, and 5000 psi) by using the parameters of Table 4 and while the permeability value is 0.1 Darcy, are compared in Figure 9.

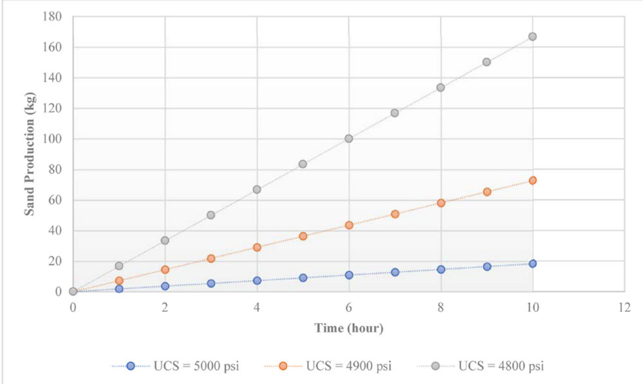

Fig. 9 Sensitivity analysis on accumulated sand production based on uniaxial compressive strength parameter.

Wellbore Pressure

As the wellbore pressure decreases, pressure drop increases, and according to Equation 20, the value of porosity will increase, and as a result, the amount of sand production will also increase. In Figure 10, using the parameters of Table 4, and while the permeability value is equal to 0.1 Darcy, the amounts of sand production are compared in three different values of wellbore pressure (1600, 1800, and 2000 psi). Reducing wellbore pressure also means increasing fluid production.

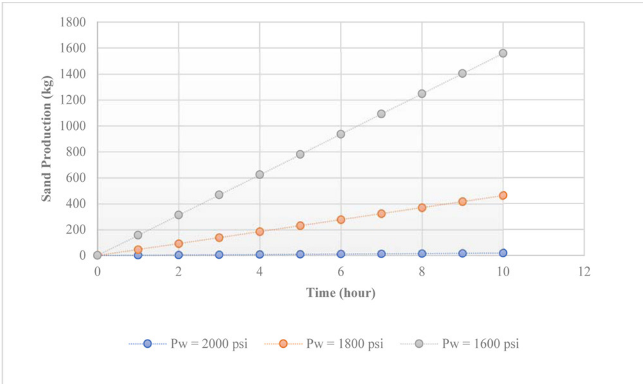

Fig. 10 Sensitivity analysis on accumulated sand production based on wellbore pressure parameter.

Maximum Horizontal Stress

As the maximum horizontal stress increases, the critical wellbore pressure increases. As a result, the critical pressure drop decreases according to Equation 20; the rate of growth in porosity value also increases, and consequently, the rate of sand production increases.

When the difference between the minimum horizontal stress and the maximum horizontal stress increases, the tangential stress also increases. Consequently, the difference between tangential stress and radial stress increases, which leads to an increase in the probability of failure.

In Figure 11, it is shown that with increased maximum horizontal stress, the amount of sand production also increases.

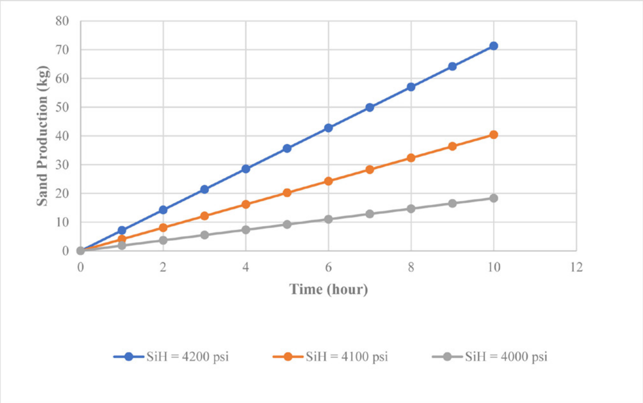

Fig. 11 Sensitivity analysis on accumulated sand production based on maximum horizontal stress parameter.

Other parameters in Table 3, such as reservoir thickness, reservoir radius, failure angle, inclination, etc., also affect sand production. For example, according to Equation 24, as the reservoir thickness increases, the amount of volume that is involved in the production of sand increases, and according to Equation 22, as the amount of volume that is involved in the production of sand increases, the amount of sand production also increases.

Conclusions

There are generally four methods for predicting sand production, including the experimental method, which they use field observations and well data, laboratory simulations, numerical methods, and analytical methods. Numerical methods are the most powerful and efficient tool for predicting sand production. This paper proposed a novel approach for the prediction of sand production by using numerical methods.

Numerical models have parameters that laboratory values or actual field values must be used to calculate those parameters and calibrate the numerical models. In this paper, by using real field data, a numerical model that is used, calibrated, and the value of sand production coefficient equal to 0.00044 s/m^3 was obtained. As shown in Figures 5 to 7, the simulated results are close to the field results, and the average relative error of the results is 6.9% which shows that the proposed model is reliable. In this model, by using computable parameters, the amount of sand production can be calculated, which it shows that this is a practical model and also the error of this model is small, because, the hypothesis parameters, are not used in this model.

Many parameters affect the amount of sand production. In this paper, using sensitivity analysis, the effect of permeability, wellbore pressure, uniaxial compressive strength, and maximum horizontal stress on the amount of sand production was investigated, and the graphs of sensitivity analysis show the reliability of the proposed model.

Nomenclatures

σ_v: Vertical stress (psi)
σ_H : Maximum horizontal stress (psi)
σ_h: Minimum horizontal stress (psi)
i: Inclination (degree)
α: Azimuth (degree)
p_w: Bottomhole pressure (psi)
p_{r_i}: Initial reservoir pressure (psi)
p_r: Current reservoir pressure (psi)
υ: Poisson ratio (dimensionless)
α: Biot coefficient (dimensionless)
β: Failure angle (degree)
C_0: Uniaxial compressive strength (psi)
ϕ_i: Initial porosity (dimensionless)
ϕ_C: Critical porosity (dimensionless)
ρ_s: Solid density (kg/m^3)
r_w: Well radius (ft)
r_e Reservoir radius (ft)
Δt: Time step (sec)
Failure model (-)
c: Concentration of solids that dissolved in the fluid (kg/m^3)
c_{cr}: Critical concentration of solids that dissolved in the fluid (kg/m^3)
ε_p: Plastic strain (dimensionless)
ε_{pcr}: Critical plastic strain (dimensionless)
V_{sp}: amount of volume that is involved in the production of sand (m^3)
r_s: Radius of the area that is changed due to sand production (ft)
k_s: Permeability of area that is changed due to sand production (Darcy)
k_{avg}: Average reservoir permeability (Darcy)
k_I: Initial reservoir permeability (Darcy)
R_{sand}: Rate of sand production (kg/sec)
Δp^{cr}: Critical drawdown pressure (psi)
Δ_p: Drawdown pressure (psi)
μ_f: Fluid viscosity (cP)
h: Reservoir thickness (ft)
k: Reservoir permeability (Darcy)
λ_{sand}: Sand production coefficient (sec/m^3)
n: Number of steps (dimensionless)

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Appendices

In Equations 27 and 28, σ_H and σ_h are the current maximum and minimum horizontal stresses, σH1 and σh1 are the initial maximum and minimum horizontal stresses, υ is the Poisson ratio, α is the poroelastic coefficient, ppi and pp are the initial and current reservoir pressures [27, 28].

$$\sigma_H = \sigma_{H1} + \frac{1-2\upsilon}{1-\upsilon} \times \alpha \times \left(p_p - p_{pi}\right) \tag{27}$$

$$\sigma_h = \sigma_{h1} + \frac{1-2\upsilon}{1-\upsilon} \times \alpha \times \left(p_p - p_{pi}\right) \tag{28}$$

In Equations 29 to 34, σ_{rr} is the radial stress, $\sigma_{\theta\theta}$ indicates the tangential stress, σ_{zz} is the axial stress, a is the distance from the center of the well, r is the radius of the well, σ_{xx}, σ_{yy} and σ_z are the initial reservoir stresses, α and i are the azimuth and the inclination angle of the well, respectively, and σ_v is the vertical stress [29, 30].

$$\sigma_{xx} = (\sigma_H \times \cos(\alpha)^2 + \sigma_h \times \sin(\alpha)^2) \times \cos(i)^2 + \sigma_v \times \sin(i)^2 \tag{29}$$

$$\sigma_{yy} = (\sigma_H \times \sin(\alpha)^2 + \sigma_h \times \cos(\alpha)^2) \tag{30}$$

$$\sigma_z = (\sigma_H \times \cos(\alpha)^2 + \sigma_h \times \sin(\alpha)^2) \times \sin(i)^2 + \sigma_v \times \cos(i)^2 \tag{30}$$

$$\sigma_{rr} = p_w \tag{31}$$

$$\sigma_{\theta\theta} = 3\sigma_{xx} - \sigma_{yy} - p_w \tag{33}$$

$$\sigma_{zz} = \sigma_z + 2\times\upsilon\times\left(\sigma_{xx} - \sigma_{yy}\right) \tag{34}$$

When σ_{xx} is less than σ_{yy}, σ_{xx} must substitute with σ_{yy} in the equations that are in Table 7. Notice that in these cases, failure initiate at $\Theta = 0^o$ and $\Theta = 180^o$. In the equations of Table 7, p_w is the critical wellbore pressure, C_0 is the uniaxial compressive strength, and β is the failure angle [31].

$$a' = 2\times c\times\cos\phi \tag{35}$$

$$b' = \sin\phi \tag{36}$$

In Equations 35 and 36, ϕ is the friction angle, and c is the cohesion.

Table 7 and Table 8 show the critical wellbore pressure relationships based on the Mohr-Coulomb failure criterion and Mogi-Coulomb failure criterion respectively.

Table 7 Critical wellbore pressure relationships based on the Mohr-Coulomb failure criterion.

Relation between stresses	Relations related to the calculation of the critical bottomhole pressure
$\sigma_{\theta\theta}>\sigma_{zz}>\sigma_{rr}$	$p_w = \dfrac{3\sigma_{xx} - \alpha p_p - \sigma_{yy} - C_0 + \left((\tan\beta)^2\right)\times\alpha p_p}{1+(\tan\beta)^2}$
$\sigma_{zz}>\sigma_{\theta\theta}>\sigma_{rr}$	$p_w = \dfrac{\sigma_z + 2\times\upsilon\times\left(\sigma_{xx}-\sigma_{yy}\right) - C_0 - \alpha p_p}{(\tan\beta)^2} + \alpha p_p$
$\sigma_{\theta\theta}>\sigma_{rr}>\sigma_{zz}$	$p_w = 3\sigma_{xx} - \sigma_{yy} - \alpha p_p - C_0 - (\tan\beta)^2\times\left(\sigma_z + 2\times\upsilon\times\left(\sigma_{xx}-\sigma_{yy}\right) - \alpha p_p\right)$

Table 8 Critical wellbore pressure relationships based on the Mogi-Coulomb failure criterion

Relation between stresses	Equations proposed for calculation of the critical bottomhole pressure
$\sigma_{\theta\theta}>\sigma_{zz}>\sigma_{rr}$	$p_w = \dfrac{Z'}{2} - \dfrac{\sqrt{3}\times\sqrt{\left(Z'-2V'+2a'+2Z'b'-4b'\times\alpha p_p\right)\times\left(2V'-Z'+2a'+2Z'b'-4b'\times\alpha p_p\right)}}{6}$
$\sigma_{zz}>\sigma_{\theta\theta}>\sigma_{rr}$	$p_w = -\dfrac{3Z'+2b'a'-K'+2V'\times b'^2-4\alpha p_p\times b'^2}{2\times\left(b'^2-3\right)}$
$\sigma_{\theta\theta}>\sigma_{rr}>\sigma_{zz}$	$p_w = \dfrac{-3Z'+2b'a'+K'+2Z'\times b'^2+2V'\times b'^2-4\alpha p_p\times b'^2}{2\times\left(b'^2-3\right)}$
$V' = \sigma_z + 2\times\upsilon\times\left(\sigma_{xx}-\sigma_{yy}\right)$	
$Z' = \left(3\sigma_{xx}-\sigma_{yy}\right)$	
The value of K' is obtained from Equation 37	

$$k' = \sqrt{\left(4Z'^2\times b'^2 - 3Z'^2 + 8Z'V'\times b'^2 + 12Z'V' - 24Z'\times\alpha p_p\times b'^2 + 12Z'\times b'a' + 16b'^2\times V'^2 - 12V'^2 - 48V'\times\alpha p_p\times b'^2 + 24V'\times b'a' + 48b'^2\times\alpha^2\times p_p^2 - 48b'a'\times\alpha p_p + 12a'^2\right)} \tag{37}$$