Abstract
Wellbore instability has always been one of the challenging issues in the drilling industry, and it could cause a delay in the drilling program, leading to an increase in the cost of the drilling projects. This study utilized data from seven wells to investigate and model directional wells’ stability in a shale formation during drilling in one of the largest oilfields in the southwest of Iran. In this study, two methods, i.e. (1) mechanical earth model (MEM) and (2) quantitative risk assessment (QRA) are applied to investigate and model directional wells’ stability in shale formation. Herein, a wellbore with full suite log data and compressional and shear wave slowness was selected to construct the mechanical earth model (MEM). Appropriate equations are provided to estimate the field’s static geomechanical data, and laboratory data were used for validation (i.e. core). The minimum mud weight required at azimuth and different angles of the directional well was calculated using numerical and analytical analysis of the wellbore stability using the Mohr-Coulomb failure criterion. The purpose of the quantitative risk assessment (QRA) phase is to investigate the impact of the uncertainty of key parameters (i.e. input variables of the minimum mud weight equation based on the Mohr-Coulomb failure criterion) and their sensitivity to an increase in success rate and a decrease in failure. In the QRA phase, the Monte Carlo simulation method is used, and the results are displayed on a Tornado diagram. The results of the Hoek-Brown and Mogi-Coulomb failure criteria propose that the sensitivity of the mud density obtained by the above methods to the uncertainty is low. The results maintain that the prediction of the minimum mud weight required for the stability of the investigated wellbore is strongly dependent on changes in the maximum horizontal stress (σ_H) parameter and minimum horizontal stress. Moreover, the internal friction angle and rock adhesion coefficient have the least impact on determining the minimum mud weight needed for wellbore stabilization.

Keywords: Wellbore stability, Mechanical earth model, Minimum and maximum horizontal stress, Quantitative risk assessment, Monte Carlo method, Tornado diagram.

Introduction
Wellbore stability is one of the major issues that drilling companies need to address during the exploration and development of hydrocarbon reservoirs to reduce costs and have safe drilling operations. The instability in our field has caused 51 wasted days of the drilling rigs. The financial loss of this time spent waiting, excluding the rig's costs for auxiliary services, is about $2,550,000. These numbers show us that mechanical instability in directional wells has created many drilling and liner driving problems. The upper and lower limit of the drilling fluid density is normally obtained based on the geomechanical model. The geomechanical model is usually derived from analytical methods and various experiments. This method is the deterministic, and assumptions are based on the characterization of in-situ stresses and rock properties with reasonable accuracy. In-situ stresses are obtained using field estimates or measurements. The information is utilized for deterministic calculations due to a lack of appropriate field data and laboratory measurement errors. But these results are highly dependent on input values. Therefore, using a deterministic approach with this level of information accuracy results in a limited classification of failure modes. These results cannot be practically employed in well drilling and completion. Error and uncertainty about inaccurate information lead to a significant impact on the final examination of wellbore stability and possibly cavity loss due to high uncertainty information. In the last decade, efforts have been made to quantify instability studies' error through probabilistic methods. Where input information is associated with uncertainty, probabilistic methods are considered very powerful methods that could reduce the
error of key parameters and increase the probability of investigation accuracy. The geomechanical model is a powerful tool to obtain a safe mud window. Insufficient reliability of the model results in an error in the stability calculations and high cost of drilling operations. Most of the uncertainty is due to the input data of the geomechanical model. Some of the parameters, for instance, pore pressure, are obtained directly, and they are highly reliable. However, some parameters, such as in-situ stresses, are not very reliable because they are obtained indirectly. Hence, this uncertainty in the input data causes an error in the geomechanical modeling process.

According to Aadnoy (2011), wellbore stability results can involve uncertainty due to inaccurate calibration [1]. Therefore, the minimum density obtained to avoid instability is associated with a high degree of uncertainty, which has been quantified. In this case, probability-based methods should be used to estimate the proper drilling fluid density and wellbore stability [1]. Statistical methods operate following QRA; that is, how far the error in the input parameters can affect the objective function and associate the results with uncertainty [2]. Much research has been done on statistical analysis in various petroleum engineering fields, including well drilling [3], specifying casing shoe, directional drilling, and formation pore pressure estimation. The effect of parameter uncertainty on wellbore stability was considered by Nawrocki, in 2010, the first researcher who studied it [4].

The statistical estimation of the lost circulation of drilling fluid using the derivative of the uncertainty estimation function was examined by him [4]. Furthermore, a model for estimating uncertainty in obtaining the minimum mud density was proposed by Colmenares et al. in 2002 [5]. Mody and Hale also obtained the minimum mud density using the Monte Carlo method and numerical simulation in 1995 [6]. The wellbore stability using uncertainty quantification (UQ) in mud density estimation was investigated by Aadnoy in 2011 [1]. Stability was investigated using QRA in a vertical well using different failure criteria by Gholami in 2014 [7]. The results of the Hoek-Brown and Mogi-Coulomb failure criteria showed that the sensitivity of the mud density obtained by the above methods to the uncertainty is low [7].

Geological Setting
The oil field investigated in this study is located in the southwest of Iran, as seen in Figure 1. The deposition of this oil field is related to the Holocene Epoch. In Table 1, the constituents in the investigated section of this field, along with lithology information, are shown. According to Table 1, the reservoir cavity mainly consists of limestone, claystone with anhydrite substrates, salt layers, and shales.

The dominant lithology of the constituent formations in the 8 ½ inch hole includes limestone, claystone with anhydrite substrates, salt layers, and shale. The studied formation has instability problems with about 5.6% shale content (volume) and an approximate thickness of 37 m. This layer containing shale has caused many irreparable problems when it has been being drilled directionally with an inappropriate trajectory and angle.

In this study, available data encompasses petrophysical logs including compressional and shear wave slowness, reservoir description tool (RDT), X-tended range micro-imager (XRMI), leak-off test (LOT), plus laboratory data of the core specifying Young’s modulus, Poisson’s ratio, and uniaxial compressive strength (UCS) of the rock.

Materials and Methods
This study initially aims to (1) calculate the rock formation’s elastic and resistive parameters, (2) estimate the pore pressure, and (3) calculate the in-situ stresses. The rock’s elastic and resistive parameters, pore pressure, and in-situ stresses are key parameters in designing the safe mud weight window (SMWW). Furthermore, key parameter estimation involves widespread uncertainty due to the lack of appropriate laboratory and field data.
Thus, in the next step, the probability distribution function (PDF) is used to apply QRA to the wellbore stability model. The purpose of applying QRA is to investigate the impact of uncertainty of key parameters on the success rate of predicting a safe mud weight window [7].

The selected well has full suite log data and compressional and shear wave slowness to investigate and model the wellbore stability. Then, geomechanical parameters for this well are calculated using basic rock mechanics equations. Afterwards, appropriate equations were selected to convert the dynamic data to static data according to formation lithology. Next, the estimation equations of these parameters are calibrated using laboratory data (i.e. core). The customized equations are presented to calculate the static geomechanical parameters in the well under study.

The formation pore pressure is one of the main input parameters of MEM construction. Thus, the formation pore pressure in the well was estimated using appropriate equations, and the results were validated by the actual data of the RDT log. Afterwards, principal overburden stress was estimated with very high accuracy using the formation density log. Principal minimum horizontal stress was estimated by existing poroelastic correlation and calibrated by leak-off test (LOT) data. Principle maximum horizontal stress was also predicted using the elastic method and calibrated by the wellbore breakout location (i.e. enlargement) at XRMI. In this part of the research, the key MEM parameters are computed.

Now, MEM is designed and prepared. However, the issue is the application of estimation equations and the existence of uncertainty in each equation. In the QRA phase, the aim is (1) to investigate the impact of the uncertainty of key parameters and their sensitivity to increasing success rates (i.e. safe drilling) and (2) to reduce failure (i.e. decreasing days of rig time and daily rig cost).

### Quantitative Risk Assessment

In the quantitative risk assessment (QRA) method, error and uncertainty are first measured in the wellbore stability model’s input data. Using the analytical concepts of the analytical functions, the wellbore stability is measured at different key parameters as a function of the hydrostatic pressure applied to the wellbore (i.e. the drilling fluid weight). The systematic process in QRA of wellbore stability involves the following steps:

1. **Step One**: Identification of key parameters and failure criteria along with measurement of error and uncertainty;
2. **Step Two**: Formulation of limit state function (LSF) to measure sensitivity and calculate surface response for each value of drilling fluid weight;
3. **Step Three**: Application of QRA for each parameter using the Monte Carlo method;
4. **Step Four**: Calculation of the probability distribution function (PDF) of all parameters with uncertainty;
5. **Step Five**: Drawing success rate logs as a function of drilling fluid weight.

In step 5, the thresholds are determined to prevent wellbore collapse and fracturing (i.e. lost circulation). The main model inputs are measured by PDFs using numerical methods such as Monte Carlo, error, and uncertainty. Hence, the input parameters with the highest uncertainty are specified. Now, the main variables (i.e. parameters) with the most significant impact on the wellbore stability should be considered by paying attention to collecting the necessary modeling data for effective risk assessment to increase the success rate and reduce time cost [8].

### Monte Carlo Simulation

In this study, Monte Carlo simulation is applied in the QRA. The Monte Carlo method (or Monte Carlo simulation) refers to any technique that approximates answers to quantitative problems through statistical sampling. Monte Carlo simulation is mostly used to describe a way to propagate uncertainties at the model’s input to uncertainties at the model’s output. Monte Carlo, then, is a simulation that explicitly and quantitatively shows uncertainty. Monte Carlo simulation relies on the process of explicitly displaying uncertainty by designating inputs as probability distributions. Suppose the inputs describing a system are uncertain. In that case, predicting the lead’s performance is necessarily uncertain. It means that the result of any analysis based on the inputs displayed with the probability distributions is itself a probability distribution. Since the result of simulating an uncertain system is a conditional report, the result of a probabilistic (Monte Carlo) simulation is a conditional probability. To calculate the predicted efficiency probability distribution, it is necessary to transfer the input uncertainties to the output uncertainties. There are various ways to convey
uncertainty. In the Monte Carlo simulation, the whole system is executed many times (1000 times). Any simulation is called system realization. All non-deterministic parameters are sampled (i.e. a random value of each parameter’s specific distribution is selected). The system is then simulated over time (with a definite set of input parameters). The simulation is performed so that the system performance can be calculated, and many independent results are produced, each of which represents a possible “future” for the system (a possible path that the system is likely to follow over time). The results of independent system realizations are transformed into possible distributions of possible outputs. As a result, the outputs are not single values, but they are probability distributions.

Results and Discussion

Dynamic elastic moduli were calculated to obtain the investigated wellbore geomechanical parameters using compressive and shear wave velocity data. They were then converted to static values and validated by lab data, including Young’s modulus, Poisson’s ratio, and uniaxial strength of the rock in the well-being studied. One of MEM construction’s most principal parts is the proper estimation of pore pressure [9]. In the well, a suitable method was selected to estimate the formation pore pressure after much investigation, and the pore pressure estimation model was validated using real data from the reservoir description tool (RDT Halliburton Tool), yielding good results. Next, the principal stresses were calculated, and then the horizontal stress was validated by the actual amount of LOT data. The maximum horizontal stress along the wellbore breakout locations was determined by analyzing XRMI logs using Techlog software.

In Table 2, the physical properties of the subjected formation are provided.

Table 2 Physical properties of the formation with wellbore instability.

<table>
<thead>
<tr>
<th>Fluid type</th>
<th>NGR(%)</th>
<th>Shale volume(%)</th>
<th>water saturation(%)</th>
<th>Porosity(%)</th>
<th>Permeability(mD)</th>
<th>Thickness(m)</th>
<th>Lithology</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil</td>
<td>68</td>
<td>5.6</td>
<td>14</td>
<td>18</td>
<td>17.8</td>
<td>37</td>
<td>Limestone-Shale</td>
</tr>
</tbody>
</table>

Mechanical Earth Model (MEM)

Following the investigations, the best equations were introduced to calculate static Young’s modulus (equation (1)), Poisson’s ratio (equation (2)), and uniaxial strength of the rock (equation (3)) in the desired vertical well formation (A)[10]. Equations (1-3) were validated using the core laboratory data available in the adjacent well (B) (Figure 3), the results of which are shown in Figure 4.

\[
E_s = 0.4145E_d - 1.0593
\]  
\[
\nu = 0.5\left(\frac{\Delta t_s}{\Delta t_c}\right)^2 - 2\left[\left(\frac{\Delta t_s}{\Delta t_c}\right)^2 - 1\right] \tag{2}
\]  

Fig. 2 Variations in shale volume in vertical well (A).
Poisson’s ratio, compressional wave slowness ((μs / ft)), shear wave slowness ((μs / ft)), the uniaxial compressive strength of the rock (kPa), neutron log values, bulk modulus (GPa), and shear modulus (GPa), respectively.

The pore pressure parameter is considered one of MEM construction’s key parameters and is usually estimated using well log analysis. Equation (4) is presented to estimate the formation pore pressure in well (A) of the desired oil field. Figure 3a shows the validation results from the actual RDT data for well (A), indicating the estimated pore pressure values’ accuracy.

$$\text{PPG} = \text{OBG} - (\text{OBG} - \text{Png}) \times \frac{(\text{Ln}_0 - \text{Ln})}{\text{cZ}}$$  (4)

where PPG is the pore pressure gradient of the formation (in Psi/ft.), OBG is the overburden pressure gradient of the formation (in Psi/ft.), Png is the normal pore pressure gradient, which is, on average, 0.45 (in Psi/ft.). According to the analysis of formation fluid and degree of salinity, $\phi$ is the formation porosity, $Z$ is the depth of the well (in ft.), $\phi'$ is the initial porosity of the formation, and $c$ is the constant value obtained from the exponential depth-porosity plot, whose values are 0.7227 and 0.0002, respectively.

Principal stresses have been calculated using Equations (5-7) [9]. The regional stress regime is normal ($\sigma_h \leq \sigma_H \leq \sigma_V$).

$$\sigma_\sigma = \int_0^x p(z) gdz$$  (5)

$$\sigma_h = \frac{v}{1-\nu} \sigma_v - \frac{v}{1-\nu} \alpha P_p + \frac{E}{1-\nu^2} e_x + \frac{v E}{1-\nu^2} e_y$$  (6)

$$\sigma_H = \frac{v}{1-\nu} \sigma_v - \frac{v}{1-\nu} \alpha P_p + \frac{E}{1-\nu^2} e_x + \frac{v E}{1-\nu^2} e_y$$  (7)

where $\sigma_h$ and $\sigma_H$ are the minimum and maximum horizontal stresses, $v$ is Poisson’s ratio, $\alpha$ is the Biot factor, $E$ is Young’s modulus, $e_x$ and $e_y$ are the strains along the minimum and maximum horizontal stresses, respectively.

Figure 3: Position of vertical wells (A and B) and directional wells (C, D, E, and F) in the oil field.

Figure 4: A: Young’s modulus graph. B: Poisson’s ratio. C: uniaxial strength of the rock. in vertical well (A) and validation of core laboratory data in adjacent well (B).

Figure 5: A) Formation pressure estimation plot along with validation of core laboratory data. B) Principal stresses plot along with the validation of real LOT data. C) MEM. D) XRMI (image log) along with wall collapse locations in vertical well (A).
The graph (Figure 5d) consists of two sections completely covering a 8 ½ inch hole in the vertical well (A). Forty-three breakout (BO) with a 161-341° trajectory in the first part (Figure 6a) and 46 breakouts in the second part of the wellbore XMI of well (A) (Figure 6b) were precisely specified to determine the trajectories of the principal horizontal stresses using Techlog.

The stress regime of the study area is normal, with the largest principal stress (σ_1) being the overburden stress along the vertical trajectory, the intermediate principal stress (σ_2), or the horizontal stress maximum along the 75-245° trajectory, and the smallest principal stress (σ_3) is horizontal stress at least along the 165-345° trajectory. Hence, drilling along the minimum horizontal stress as the optimal path requires the least mud weight. The optimal directional drilling trajectory’s mere determination will not solve this problem because the azimuth of directional wells is determined by the employer and according to production schedules. Therefore, numerical and analytical analyses are performed to determine the minimum mud weight needed to stabilize the directional well at each azimuth, and well angle is calculated by the Mohr-Coulomb failure criterion. In Figure 7, the results of this model are shown.

QRA is defined as equation (8):

\[ F_L = F_C(X) - F_L(X) \]  

where F is the failure function, FC is the critical failure function, and FL is defined as LSF. \( x \), as an independent variable, is a random variable that is the same key parameters used in the construction of MEM in wellbore stability analysis. Thus, the minimum mud weight required to stabilize the wellbore is obtained using the Mohr-Coulomb

\[ 3σ_H - σ_h ≥ \frac{2C \cos(φ) + Pp (1+\sin(φ) - 1)}{1+Sin(φ)} \]

According to equation (9), the input parameters needed to determine the PDF are \( σ_H \) (maximum horizontal stress), \( σ_h \) (minimum horizontal stress), \( φ \) (internal friction angle of the rock), \( c \) (rock adhesion), and \( Pp \) (formation pore pressure) is given as equation (10). The uncertainty of each of the key parameters is examined separately, and the most appropriate PDF and the adjustment curve are selected to normalize and maximize data coverage.

\[ P_{wMC}(X) = P(σ_H, σ_h, φ, c, Pp) \]  

As stated above, the five main parameters as input variables are independent of the status limit. These independent variables are \( σ_H \) (maximum horizontal stress), \( σ_h \) (minimum horizontal stress), \( φ \) (internal friction angle of the rock), C (rock cohesion), and \( Pp \) (pore formation pressure). The uncertainties of each of the main parameters are examined separately, and the most appropriate probability distribution functions are shown in Figures 8 to 12. Diagrams of red bars corresponding to the main parameter’s probability distribution and the distribution function (blue curve) are presented here.

Analysis of Directional Wellbore Stability by QRA

In the QRA method, the input data of the model are first evaluated and measured numerically. Afterwards, the wellbore stability is measured as a function of the pressure applied to the wellbore (i.e. the drilling fluid) weight by 3D models using probability analytical concepts. This study used nonlinear elastic concepts. The status limit function in
Fig. 10 Weibull probability distribution function matching the original FANG parameter.

Fig. 11 Normal probability distribution function matching the main parameter C.

Fig. 12 Log Logistic probability distribution function matched to the original PP parameter.

In the Monte Carlo simulation, the results of independent system investigations are transformed into probability distributions of possible outputs. As a result, the outputs are not as single values but as probability distributions. In this study, the QRA of the minimum mud weight required for wellbore stability (A) was simulated numerically using the Monte Carlo method. Figure 13 shows the results of the sensitivity analysis by the Tornado diagram.

According to the Tornado diagram shown in Figure 13, the minimum horizontal stress, internal friction angle, and rock adhesion coefficient are the parameters which only have the least impact on determining the minimum mud weight needed to stabilize the wellbore in the field under consideration. Therefore, the uncertainty in estimating these parameters is not of great importance. For example, 14% of the variation in minimum horizontal stress values ($\sigma_h$) can make up to 21% of the difference in estimating the minimum mud weight needed for wellbore stability.

Nevertheless, the prediction of the minimum mud weight required for the stability of the investigated wellbore is strongly dependent on changes in the maximum horizontal stress parameter ($\sigma_v$). Hence, the slightest error in measuring the maximum horizontal stress value can lead to many errors in determining the minimum mud weight required for wellbore stability.

Making about 17.5% of the variation in the $\sigma_v$ value results in changes in the required minimum mud weight up to 84%. It is not possible to directly determine the maximum horizontal stress, which causes a great deal of uncertainty in estimating this parameter. As shown in Figure 14, the Mohr-Coulomb failure criterion of the safe window (without wall collapse and lost circulation of the formation) estimates that the mud weight is between 1.3 and 1.5 (g/cm$^3$) with a success rate of 75%.

Conclusions

In this study, two modeling techniques were performed: (1) mechanical earth modeling and (2) QRA modeling. In general, the following results were obtained from this study:

1. MEM outputs include values of principal stresses. Principal stresses in the vertical well (A) were calculated using elastic modulus and elastoplastic equations. Afterwards, the regional stress regime was determined as normal ($\sigma_1 \leq \sigma_\parallel \leq \sigma_3$).

Therefore, drilling along the minimum horizontal stress, as the optimal path, requires the lowest mud weight and vice versa (this means that drilling along the maximum horizontal stress requires the highest mud weight).
2. The prediction of the minimum mud weight required for the stability of the investigated wellbore is strongly dependent on changes in the maximum horizontal stress ($\sigma_H$) parameter. Hence, the slightest error in measuring the maximum horizontal stress value can cause many errors in determining the minimum mud weight required for wellbore stability. Minimum horizontal stress, internal friction angle, and rock adhesion coefficient are among the parameters with the least impact on determining the minimum mud weight needed to stabilize the wellbore in the field under consideration. Therefore, the uncertainty in estimating these parameters is of no great importance.

3. An imprecise calculation, about 17%, in $\sigma_H$ value causes an 84% change in minimum mud weight required, and a 14% change in minimum horizontal stress ($\sigma_h$) values can make up to a 21% difference in estimating the minimum mud weight required for wellbore stability.

4. Ultimately, the Mohr-Coulomb failure criterion of safe window estimates mud weight between 1.3 and 1.5 (g/cm$^3$) with a 75% success rate.

Nomenclatures

BO: Breakout
Lot: Leak-off test
MEM: Mechanical earth model
PDF: Probability distribution function
QRA: Quantitative risk assessment
RDT: Reservoir description tool
SMWW: Safe mud weight window
XRMI: X-tended range micro-imager

References