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Water Flooding Performance Evaluation Using Percolation Theory

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Abstract
Water flooding is a well-known secondary mechanism for improving oil recovery. Conventional approach to evaluate the performance of a water flooding process (e.g. breakthrough and post breakthrough behavior) is to establish a reliable geological reservoir model, upscale it, and then perform flow simulations. To evaluate the uncertainty in the breakthrough time or post breakthrough behavior, this procedure has to be repeated for many realizations of the geological model, which takes many hours of CPU time. Moreover, during the early stage of reservoir life, when data is scare, breakthrough and post breakthrough time behavior prediction are usually based on analogues or rules of thumb. Alternative statistical approach is to use percolation theory to predict breakthrough and post breakthrough bahavior. The main contribution is to evaluate the applicability of the existing scaling laws of the breakthrough time by the numerical flow simulation results using the Burgan formation dataset of Norouz offshore oilfield in the south of Iran. Moreover, we extend the scaling to the post breakthrough behavior. There is good agreement between the predictions from the percolation based expressions and the numerical simulation results. Moreover, the prediction from the scaling law took a fraction of a second of CPU times (as it only needs some algebraic calculations) compared with many hours required for the conventional numerical simulations.

Key words: Percolation, Breakthrough time, Post breakthrough behavior, Validation, Case study.

Introduction
The breakthrough time, t_{br} , accounts for the first passage time (i.e. transport) between the injector and the producer. The knowledge of this time has many important applications. Consider water flooding as a secondary recovery method. Once the injected fluid (i.e. water) breaks through in the production wells, the fraction of extracted fluid (i.e. oil) will be reduced. It is important to know when the injected fluid will breakthrough and what the rate of decline in oil production will be for the economic evaluation of an upstream project.

The conventional approach to predict the breakthrough time or post breakthrough behavior is to build a detailed geological model, upscale it, and then perform flow simulations. To estimate the uncertainty in such a system, this has to be repeated for various realizations of the geological model. The problem with this approach is that it is computationally very expensive and time consuming. Although there has been some progress to reduce the calculation time [1, 2], there is a great incentive to produce much simpler physically based techniques to predict these reservoir performance parameters very quickly, particularly for engineering purposes.

In this paper, we use percolation based reservoir structure [3] on which we can analyze the breakthrough time and post breakthrough behavior between an injector and a producer [4]. There are many interesting cases where the geological formations consist of a mixture of good sandstones with high permeability (i.e. flow units) and poorer siltstones, mudstones and shales with negligible permeability. Good sandstones with significant permeability and porosity often contain the majority of formation fluid (e.g. oil). Typical examples are fluvial sediments containing paleochannels (highly pemeable zones embedded in a low permeable background), shale/sandstone sequences (impermeable inclusions embedded in a permeable matrix), fractured formations (with fractures as connected high permeable zones), and coastal depos its (deltaic systems representing the permeable media).

Percolation concepts can be used to find the probability distribution of the breakthrough time between an injector and a producer. Dokholyan et al. [5] did the pre-elementary study of the breakthrough time. They presented a scaling ansatz for the distribution function of the shortest paths connecting any two points on a percolating cluster. Traveling time and traveling length for tracer dispersion in two dimensional bond percolation systems have been studied by Lee et al. [6]. King et al. [7] used percolation theory to predict the distribution of the shortest path between well pairs and presentd a scaling hypothesis for this distribution, which has been confirmed by the numerical simulation. Andrade et al. [4] concentrated on the flow of fluid between two sites on the percolation cluster. They modelled the medium by using bond percolation on a lattice, while the flow front was modeled by tracer particles driven by a pressure difference between two fixed sites representing the injection and productions wells. They investigated the distribution function of the shortest path connecting these two wells and proposed a scaling ansatz, which was confirmed by extensive simulations. Moreover, Andrade et al. [8] investigated the dynamics of viscous penetration in two dimensional percolation networks at criticality for the case in which the ratio between the viscosities of displaced and injected fluids is very large. They reported extensive numerical simulations showing that the scaling exponents for the breakthrough time distribution was the same as the previously reported values computed for the case of unit viscosity ratio. Araujo et al. [9] analyzed the distributions of traveling length and minimal traveling time through two dimensional percolation porous media characterized by relatively long range spatial correlations. They found that the probability distribution functions follow the same scaling ansatz originally proposed for the uncorrelated case, but with quite different scaling exponents. Using Monte Carlo simulations, Paul et al. [10] determined the scaling form for the probability distribution of the shortest path between two lines representing the wells in a three dimensional percolation system at criticality, where the two wells could have arbitrary positions, orientations, and lengths. Comparing the result of the proposed scaling laws from percolation theory with the time for a fluid injected into an oilfield to breakthrough into a production well was studied by King et al. [11]. Lopez et al. [12] numerically simulated the traveling time of a tracer in convective flow between two wells in a percolation porous media. They analyzed the traveling time probability density function for two values of the fraction of connecting bonds, namely the homogeneous case and the inhomogeneous critical threshold case. Soares et al. [13] investigated the distribution of shortest paths in percolation systems at the percolation threshold from one injector well to multiple producer wells. Lopez et al. [14] studied the behavior of the optimal path between two sites on a d-dimensional lattice with weight assigned to each site. They calculated the probability distribution of the optimal path length. Li et al. [15] applied percolation method to estimate interwell connectivity of thin intervals for non-communicating stratigraphic intervals in an oilfield.

The aim of this study is to characterize the breakthrough time and post breakthrough behavior between an injector and a producer by using the scaling law propsoed by Andrade et al. [4] against the detailed reservoir simulation results. For validation, we use the Burgan reservoir dataset of Norouz offshore oilfield in the south of Iran. Furthermore, we developed a scaling law for the prediction of post breakthrough behavior based on the probability distribution of breakthrough time. For this purpose, we focus on the time to reach the water cut of 50% in the production well and introduce a new probability distribution function for this.

Breakthrough time

Preliminary studies of breakthrough time behavior [4, 5, 6, 9, and 11] indicate that the percolation model gives useful predictions. It has been shown that the distribution of breakthrough time conditioned to the formation size, L, and net to gross ratio, p, has the scaling [4]:

$$P\left(t_{br} \mid r, L, p\right) \propto \frac{1}{r^{d_t}}\left(\frac{t_{br}}{r^{d_t}}\right)^{-g_t} f_1\left(\frac{t_{br}}{r^{d_t}}\right) f_2\left(\frac{t_{br}}{L^{d_t}}\right) f_3\left(\frac{t_{br}}{\xi^{d_t}}\right) \quad (1)$$

Where $f_1(x)=e^{-ax^{-\phi}}$, $f_2(x)=e^{-bx^{\psi}}$, $f_3(x)=e^{-cx^{\pi}}$, t_{br} is the breakthrough time for a given realization, r is the Euclidean distance between the injector and a producer, and   is the typical size of reservoir sandbodies. The numerical values of the other exponents were determined through rigorous simulation studies (table 1).

Table 1. Numerical values of the exponent in Eq. 1[4]

Exponent	Value	Exponent	Value
d_t	$2D$: 5.0 $3D$: 2.3	c	$2D$: 1.6(−), 2.6(+) $3D$: 2.9(−)
g_t	$2D$: 5.0 $3D$: 2.3	Φ	$2D$: 3.0 $3D$: 1.6
a	$2D$: 5.0 $3D$: 2.3	ψ	$2D$: 3.0 $3D$: 2.0
b	$2D$: 5.0 $3D$: 2.3	π	$2D$: 1 $3D$: 1

Eq. 1 assumed that the injecting and producing fluids are incompressible; that is, the displacing fluid has the same viscosity and density as displaced fluid (i.e. like passive tracer transport). Therefore, as the injected fluid displaces the oil, the pressure field is unchanged. Pressure field is determined by the solution of the single phase flow equation (i.e. Laplace equation, $\nabla(K\nabla P)=0$). In addition, the injected fluid follows the streamlines, which are normal to the isobars.

Fixed pressure boundary conditions were considered at the two wells. For each possible configuration of the system, there are many possible streamlines from the injector to the producer well where the injected fluid is passively convected along these streamlines. Breakthrough time corresponds to the time when the first tracer reaches the production well for a given reservoir realization.

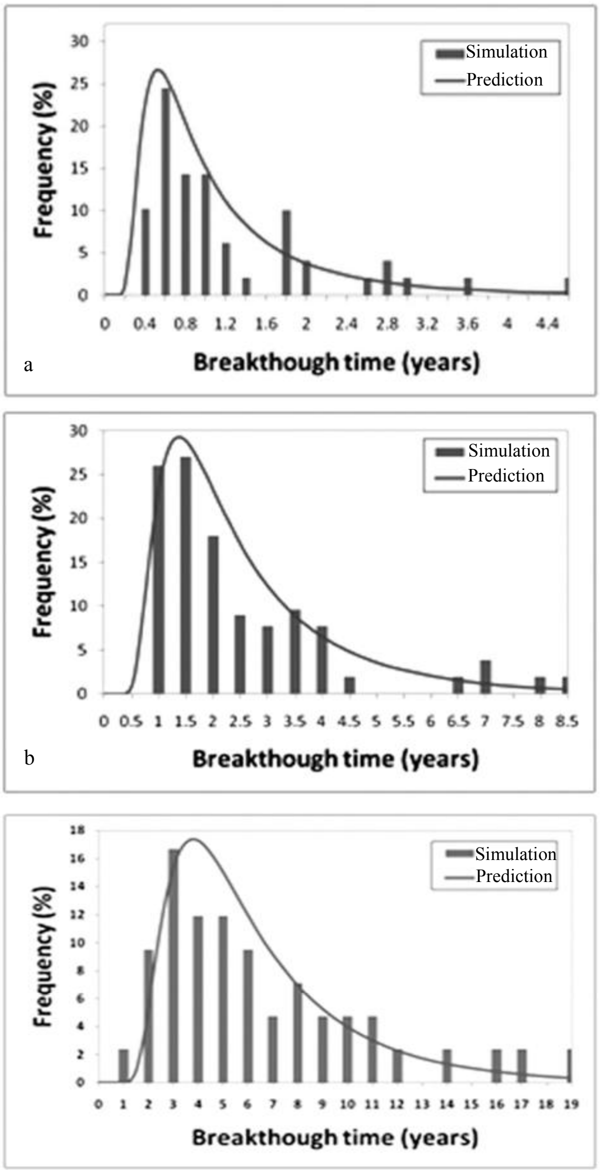
The probability function given by Eq. 1 can be encoded in a spreadsheet from which, using some primary data, the probability distribution of the breakthrough time is determined very quickly. The motivation behind developing Eq. 1 is that in a disordered medium, the streamlines are like self avoiding random walks with the probability distribution of stretched Gaussian like function, f_1. Accounting for finite size effects, large excursions of the streamlines are not permitted by finite boundaries so there will be an upper cut-off given by function f_2. In addition, to account for the size of finite size clusters away from the threshold, the truncation of the excursion of the streamlines given by function f_3 must be considered.

Application to real field

To validate the approach, we have used the Burgan formation dataset of Norouz offshore oilfield in the south of Iran. Core and palynological data indicated that the Burgan consists of a series of incision fill sequences occurring in an estuarine/coastal plain/deltaic environment. Consequently, the Burgan formation consists of a thick stack of excellent quality sands incising into each other with a few remaining shalier sediments locally separating these sequences. The excellent formation sands of the Burgan consist mainly of valley fill deposition sediments, where freshwater fluvial, coarse grained sediment (highly permeable zones) accumulated in the upper and middle reaches of the incised valley system, while tidally influenced sediments accumulated in the marine influenced middle and lower reaches. As a result of these aggradations, less fluvial derived sediment reached the lower reaches of the valley system, which resulted in the clean, well sorted shoreline material being reworked and deposited in the tidal channels in this area [16]. Because of this process, the valley fill deposition sediments can be considered as high permeable flow units in a background that is essentially impermeable, which makes the Burgan reservoir ready to be modeled with the percoaltion approach.

Validation of the breakthrough scaling law

To have different realizations, various points for the well locations in the entire Burgan reservior were randomly selected. Nearly 300 flow simulations were run on these wells at the full scale of the the Burgan reservoir formation. To include well spacing effect, three distinct well spacings are considered as 500m, 750m, and 1000m. Afterward, from the simulation results, the probability distributions of the breakthrough time for each of these well spacings were determined. The comparison of the result of the scaling law of the breakthrough time, Eq. 1, (dashed curves) with the results obtained from the conventional numerical simulations (the bar chart) for three well spacing is shown in Fig. 1.



c

Figure 1. Comparison of prediction of the breakthrough time using the scaling laws of Eq. 1 (curve) with results from the numerical simulations (bar chart) for well spacing of a) 500m, b) 750m, c) 1000 m.

As it can be seen in Fig. 2, there is reasonable agreement between the prediction from the scaling law, given by Eq. 1, and the direct numerical simulation results. Moreover, the prediction from the scaling law took a fraction of a second of CPU times (as it only needs some algebraic calculations) compared with hours required for the conventional numerical simulations.

Post breakthrough behavior

The main question in evaluating the post breakthrough behavior is that if the probability distribution of the breakthrough time is available, how this reduces the uncertainty in the post breakthrough behavior.

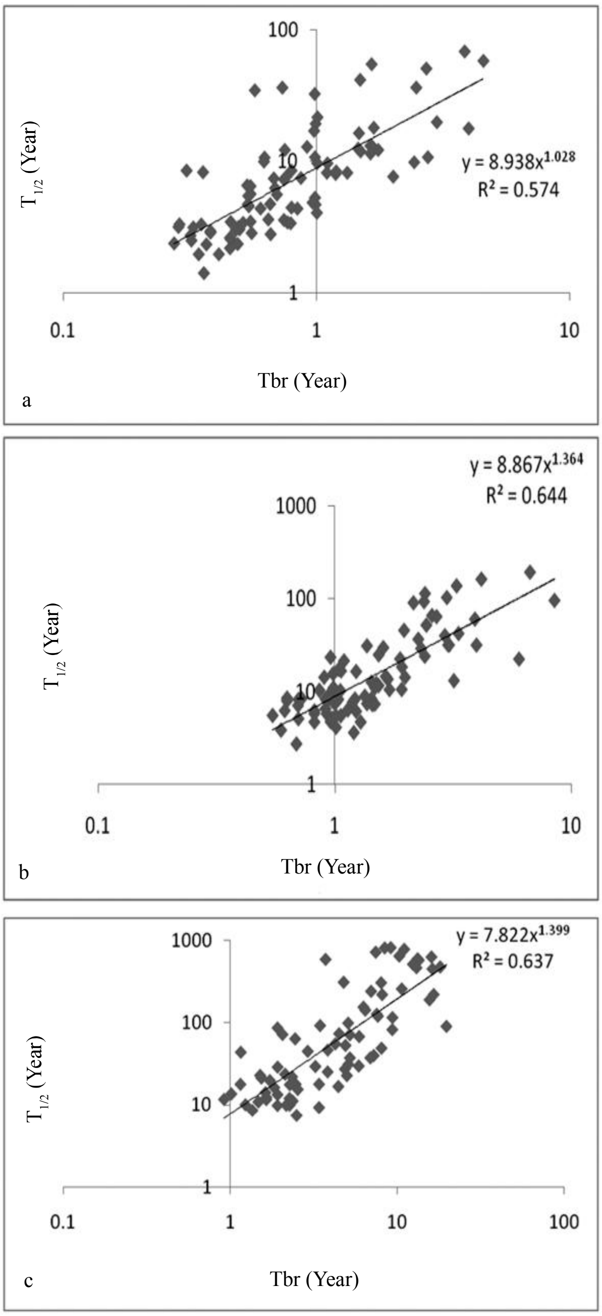


Figure 2. Illustration of time to reach the water cut of 50% in production wells versus breakthrough time for different well spacing as a) 500m, b) 750m, c) 1000 m.

Specifically, we want to check if there is a correlation between the breakthrough time results and the time taken for the oil production to fall by for example 50%, so called $t_{1/2}$, (or water cut to increase to 50%). To get these statistics, we continue the conventional flow simulations to reach this water cut value. Then, we plot $t_{1/2}$ versus t_{br} on a log-log scale. Fig. 2 shows the results of these cross plots at different well spacings values.

As it can be seen, there is a correlation between the breakthrough time and the time to reach the water cut of 50% in the production well. By plotting the result of all simulation results from different well spacing (Fig. 3), one can check the generality of the relationship between these two times.

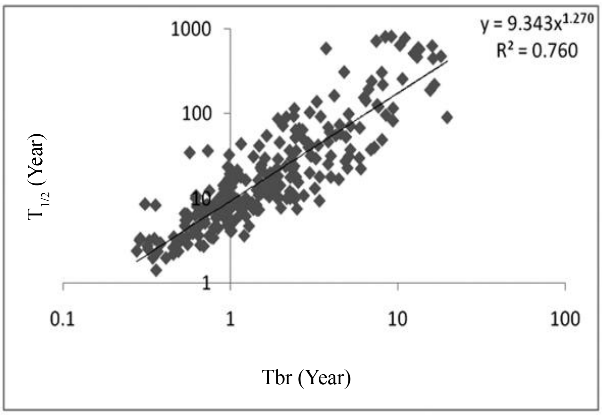


Figure 3. Illustration of the time to reach the water cut of 50% in production wells versus breakthrough time

The general relation between the two times $t_{1/2}$ and t_{br} or the Burgan reservoir is as follows:

$$t_{1/2} = 9.343 t_{br}^{1.27} \tag{2}$$

Therefore, for estimation of the probability distribution of the post breakthrough time (defined by the water cut of 50%), one can use the probability distribution of the breakthrough time from Eq.1 and scale it by using Eq. 2 (Fig. 4a). Figure 4b compares the simulation results of the post breakthrough behavior with the proposed scaling law.

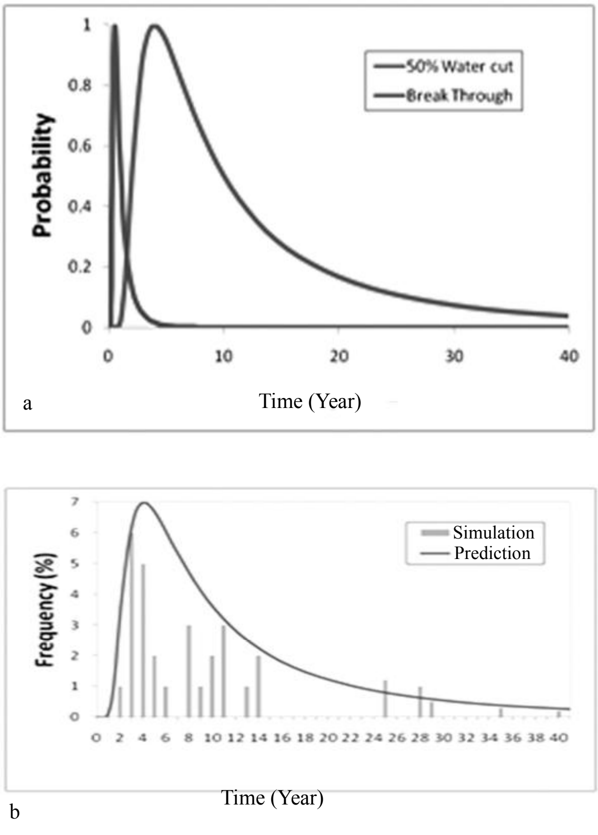


Figure 4. a) Scaling anstaz for the probability distribution of the post breakthrough behavior in comparison with the probability of breakthrough time **b)** Comparison of prediction of the time to reach the water cut of 50% using the scaling laws of post breakthrough behavior (curve) with results from the numerical simulations (bar chart)

Conclusion and Recommendations

Percolation theory is a powerful theory, which can be used for prediction of field scale properties of reservoirs. We set out to estimate the breakthrough time and post breakthrough behavior between two injection and production wells in terms of percolation concepts. In particular, we have checked that percolation approach could be used to help produce results of interest for engineers without having to perform a large number of numerical simulations for each new geological media even at the early stage of reservoir life when the availability of certain data is questionable. In particular, the proposed scaling law of the breakthrough time has been compared with the results obtained from the conventional numerical simulations on the Burgan formation dataset of Norouz offshore oilfield in the south of Iran. In addition, we set out to find a scaling law for the post breakthrough behavior using the probability distribution of the breakthrough time. There was good agreement between the predictions from the percolation based expression and the numerical simulation results. Moreover, the prediction from the scaling law took a fraction of a second of CPU times (as it only needs some algebraic calculations) compared with many hours required for the conventional numerical simulations.

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