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FORCED CONVECTION HEAT TRANSFER OF GIESEKUS VISCOELASTIC FLUID IN CONCENTRIC ANNULUS WITH BOTH CYLINDERS ROTATION

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ABSTRACT

A theoretical solution is presented for the forced convection heat transfer of a viscoelastic fluid obeying the Giesekus constitutive equation in a concentric annulus under steady state, laminar, and purely tangential flow. A relative rotational motion exists between the inner and the outer cylinders, which induces the flow. A constant temperature was set in both cylinders, in this study. The fluid properties are taken as constants and axial conduction is negligible, but the effect of viscous dissipation is included. The dimensionless temperature profile, the normalized bulk temperature, and the inner and outer Nusselt numbers are derived from solving non-dimensional energy equation as a function of all relevant non-dimensional parameters. The effects of Deborah number (*De*), mobility factor (α), Brinkman number (*Br*) and velocity ratio (β) on the normalized temperature profile and Nusselt number are investigated. The results indicate the significant effects of these parameters on the dimensionless temperature distribution and Nusselt number.

Keywords: Viscoelastic Fluid, Giesekus Model, Elasticity, Viscous Dissipation, Nusselt Number

INTRODUCTION

Flow induced by a relative rotational motion between two cylinders in a concentric annulus has many significant engineering applications such as rotating electrical machines, swirl nozzles, rotating disks, standard commercial rheometers, and other chemical and mechanical mixing equipment [1]. Moreover, the problem of heat transfer in annulus has practical importance in engineering applications. For example, polymer processing generally consists of three steps: melting of plastic, shaping, and cooling to preserve the final shape. Clearly, heat transfer occurs during all the three steps of the manufacturing process and ignoring heat transfer effects may lead to substantial errors in computations.

A comprehensive review relevant to experi-menttal, numerical, and analytical research of forced convective heat transfer has been investigated by Childs and Long [2] in stationary and rotating annuli for Newtonian fluids. Khellaf and Lauriat [3] analyzed the flow and heat transfer of Carreau fluid in the annular space between two concentric cylinders where the inner cylinder is rotated and the outer cylinder is at rest. Naimi et al. [4] have investigated forced convection of the Taylor–Couette vortices for the case of a power-law fluid (Carbopol 940) with and without axial flow. Research on heat

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transfer of viscoelastic fluid is scarcer. The investigation of convective heat transfer visco-elastic fluid inside a concentric annulus for axial flow was performed analytically by Pinho and Coelho [5] using Simplified Phan-Thien-Tanner (SPTT) model. Their solutions were presented for both imposed constant wall heat fluxes and imposed constant wall temperatures, always taking into account viscous dissipation. Mirzazadehet et al. [6] presented an analytical solution for convective heat transfer of a viscoelastic fluid obeying Phan-Thien-Tanner (PTT) model in the presence of viscous dissipa-tion in a concentric annulus with a relative rotation of the inner and outer cylinders.

For viscoelastic Giesekus model a theoretical solution was presented in pipe and channel by Mahdavi et al. [7]. Their study included the effect of viscous dissipation and the constant heat flux imposed at the wall.

This paper will presents an analytical solution of forced convection heat transfer for viscoelastic fluid obeying Giesekus model for purely tangential flow, including viscous dissipation and imposed constant temperature at walls in concentric annulus. The effects of elasticity, mobility parameter, and viscous dissipation on the temperature distribution and Nusselt number will be discussed. The extensive details of this model have been presented elsewhere [8, 9].

Mathematical Formulation

The configuration of the problem studied in this paper is depicted in Figure 1. The flow is considered to be steady state, laminar, and purely tangential flow in a concentric annulus. A relative rotational motion exists between the inner and outer cylinders, which induces the flow. The ratio of outer cylinder radius (R_o) to inner cylinder radius (R_i) is defined as Π, also the annular gap (δ) equals to (Π-1). Fluid properties and model parameters are considered independent of temperature. Fluids found in polymer processing (polymer melts and concentrated solutions) are usually very viscous and industrial flows frequently involve large velocity gradients; thus viscous dissipation effects can be important and therefore will be taken into account; an isothermal condition is considered in this study.

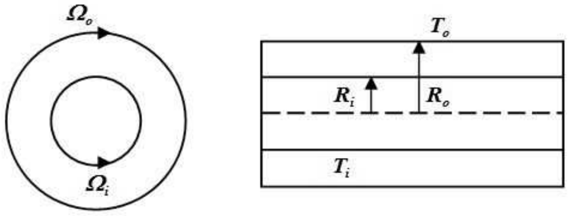

Figure 1: Schematic diagram of the problem under consideration

The energy equation by assuming viscous dissipation and neglecting axial heat conduction can be presented by the following equation:

$$\frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\tau:\nabla V=0 \quad (1)$$

where, k is thermal conductivity and T is the fluid temperature.

The viscous dissipation term ($\tau.\Delta V$) is given by:

$$\tau:\nabla V=\tau_{r\theta}\frac{\partial V_\theta}{\partial r}-\tau_{r\theta}\frac{V_\theta}{r} \quad (2)$$

The boundary conditions are:

$$r=R_i \qquad V_\theta=R_i\Omega_i \qquad T=T_i \quad (3)$$

$$r=R_o \qquad V_\theta=R_o\Omega_o \qquad T=T_o \quad (4)$$

By employing non-dimensional quantities, energy equation can be written as reads:

$$\frac{1}{R^*}\frac{\partial}{\partial R^*}\left(R^*\frac{\partial\Theta}{\partial R^*}\right)+\frac{Br}{\Pi-1}\tau_{r\theta}^*R^*\frac{\partial}{\partial R^*}\left(\frac{V_\theta^*}{R^*}\right)=0 \quad (5)$$

where, $R^*=r/\delta$, $\tau_{r\theta}^*$ is dimensionless shear stress and defined as $\tau_{r\theta}/(\eta V_c/\delta)$ and V_θ^* is dimensionless velocity (V_θ/V_c). V_c is characteristic velocity ($V_C = R_i \Omega_i$). Θ stands for dimensionless local temperature and *Br* is the dimensionless Brinkman number, which is a measure of the importance of the viscous dissipation term.

Two types of normalized temperature and Brinkman number are proposed relative to different and identical temperatures on the walls.

$$\Theta = \frac{T - T_i}{T_o - T_i} \qquad Br = \frac{\eta V_c^{\,2}}{k(T_o - T_i)} \qquad T_i \neq T_o \tag{6}$$

$$\Theta = \frac{T - T_{in}}{T_w - T_{in}} \qquad Br = \frac{\eta V_c^{\,2}}{k(T_w - T_{in})} \qquad T_w = T_i = T_o \tag{7}$$

where, T_{in} represents inlet temperature.

Analytical Solution

Fluid Constitutive Equation

In this work, the Giesekus constitutive equation (without retardation time) was employed.

$$\tau + \frac{\alpha\lambda}{\eta}(\tau.\tau) + \lambda\frac{\vartheta\tau}{\vartheta t} = 2\eta D \tag{8}$$

where,

$$D = \frac{1}{2}[\nabla V + (\nabla V)^T] \tag{9}$$

$$\frac{\vartheta\tau}{\vartheta t} = \frac{D\tau}{Dt} - [\tau.\nabla V + (\nabla V)^T.\tau] \tag{10}$$

$$\frac{D\tau}{Dt} = \frac{\partial\tau}{\partial t} + (V.\nabla)\tau \tag{11}$$

τ is the stress tensor. η and λ are the model parameters representing zero shear viscosity and zero shear relaxation time respectively [9]. In addition, the parameter α in Equation 1 is mobility factor which represents anisotropic Brownian motion and/or anisotropic hydrodynamic drag on the constituent polymer molecules [10]; it is required that $0 \le \alpha \le 1$ as discussed elsewhere [8].

Hydrodynamic Solution

The hydrodynamic solution of this flow was derived by Takht Ravanchi et al. [11]. Equations 12, 13, and 14 are shear rate, velocity profile, and shear stress respectively presented in their study.

$$\dot{\gamma}^* = \frac{\Pi - 1}{\Pi} R^* \frac{\partial}{\partial R^*}\left(\frac{V_\theta^*}{R^*}\right) = \frac{1 + (2\alpha - 1)De\,\tau_{rr}^*}{(1 + De\,\tau_{rr}^*)^2}\tau_{r\theta}^* \qquad \tau_{r\theta}^* = \frac{\tau_{wi}^*}{R^{*2}} \tag{12}$$

$$\frac{V_\theta^*}{R^*} = \frac{\tau_{wi}^*}{\Pi - 1}\left(\frac{R^{*2}(\alpha - 1)}{2(R^{*2} - \alpha De^2 \tau_{wi}^{*\,2})} - \frac{\sqrt{\alpha}\arctan h\left(\frac{R^{*2}}{\sqrt{\alpha} De\,\tau_{wi}^*}\right)}{2De\,\tau_{wi}^*}\right) + c_1 \tag{13}$$

$$c_1 = 1 - \frac{\tau_{wi}^*}{\Pi - 1} \times \left(\frac{\alpha - 1}{2(1 - \alpha De^2 \tau_{wi}^{*\,2})} - \frac{\sqrt{\alpha}\arctan h\left(\frac{1}{\sqrt{\alpha} De\,\tau_{wi}^*}\right)}{2De\,\tau_{wi}^*}\right) \tag{14}$$

In these equations, *De*, the Deborah number, is defined as ($De = \lambda V_C / \delta$) and related to the level of elasticity; τ_{wi}^* can be obtained from substituting both boundary conditions in Equation 13 and solving by Newton-Raphson method.

Heat Transfer Solution

Substituting Equations 13 and 14 into Equation 5 and integrating gives the dimensionless temperature profile as follows:

$$\Theta = \frac{\sqrt{\alpha} Br\,\tau_{wi}^*}{8De(\Pi - 1)^2}\left[PolyLog\left(2, \frac{R^{*2}}{X}\right) - PolyLog\left(2, -\frac{R^{*2}}{X}\right)\right] - \frac{Br\,\tau_{wi}^*(\alpha - 1)}{8\sqrt{\alpha}De(\Pi - 1)^2} Log\left(\frac{R^{*2} - X}{R^{*2} + X}\right) + c_2 Log(R^*) + c_3 \tag{15}$$

where, $X = \sqrt{\alpha} De\,\tau_{wi}^*$ and c_2 and c_3 are constants of integration.

The dimensionless form of boundary conditions can be written as follows:

I) $T_i \neq T_o$

$$\Theta = 0 \quad \text{at} \quad R^* = 1 \tag{16-a}$$

$$\Theta = 1 \quad \text{at} \quad R^* = \Pi \tag{16-b}$$

II) $T_w = T_i = T_o$

$$\Theta = 1 \quad \text{at} \quad R^* = 1 \tag{17-a}$$

$$\Theta = 1 \quad \text{at} \quad R^* = \Pi \tag{17-b}$$

By imposing these boundary conditions in Equation 15, the constants of integration for the case of $T_i \neq T_o$ can be obtained as follows:

$$\begin{aligned} c_{3_I} = & \frac{Br\,\tau_{wi}^* (\alpha - 1)\log\left(\frac{\Pi^2 - X}{\Pi^2 + X}\right)}{8\sqrt{\alpha}De(\Pi - 1)^2 Log(\Pi)} \\ & + \frac{Br\,\tau_{wi}^* \alpha\left(PolyLog\left[2, -\frac{\Pi^2}{X}\right]\right)}{8\sqrt{\alpha}De(\Pi - 1)^2 Log(\Pi)} \\ & - \frac{Br\,\tau_{wi}^* \alpha\left(PolyLog\left[2, \frac{\Pi^2}{X}\right]\right)}{8\sqrt{\alpha}De(\Pi - 1)^2 Log(\Pi)} \\ & - \frac{c_4 - 1}{Log(\Pi)} \end{aligned} \tag{18}$$

$$\begin{aligned} c_{4_I} = & -\frac{Br\,\tau_{wi}^*}{8\sqrt{\alpha}De(\Pi - 1)^2} \times \\ & \left[(\alpha - 1)Log\left(\frac{1 - X}{1 + X}\right)\right. \\ & \left. + \alpha\left(PolyLog\left(2, -\frac{1}{X}\right) - PolyLog\left(2, \frac{1}{X}\right)\right)\right] \end{aligned} \tag{19}$$

And for the case of $T_w = T_i = T_o$, the constants are derived as $c_{3_{II}} = c_{3_I}$ and $c_{4_{II}} = 1 + c_{4_I}$.

The convective heat transfer from walls to the fluid is quantified by Nusselt number on the inner (Nu_i) and outer walls (Nu_o). This Nusselt number, based on hydraulic diameter ($D_H = 2\delta$) is defined as ($Nu = 2\delta h/k$). The heat transfer coefficient (*h*) in walls is obtained from $q_w = h(T_w - T_b)$. By using Non-dimensional temperature definition (Equations 6 and 7), the following expressions are derived for the inner and outer Nusselt numbers:

I) $T_i \neq T_o$

$$Nu_i = 2(\Pi - 1)\frac{\left.\frac{\partial \Theta}{\partial R^*}\right|_{R_i^*}}{\Theta_b} \tag{20-a}$$

$$Nu_o = 2(\Pi - 1)\frac{\left.\frac{\partial \Theta}{\partial R^*}\right|_{R_o^*}}{1 - \Theta_b} \tag{20-b}$$

II) $T_w = T_i = T_o$

$$Nu_i = -2(\Pi - 1)\frac{\left.\frac{\partial \Theta}{\partial R^*}\right|_{R^*=1}}{1 - \Theta_b} \tag{21-a}$$

$$Nu_o = 2(\Pi - 1)\frac{\left.\frac{\partial \Theta}{\partial R^*}\right|_{R^*=\Pi}}{1 - \Theta_b} \tag{21-b}$$

Θ_b is dimensionless bulk temperature which can be determined as follows:

$$\theta_b = \frac{\int_{R_i^*}^{R_o^*} V_\theta^* \theta R^* dR^*}{\int_{R_i^*}^{R_o^*} V_\theta^* R^* dR^*} \tag{22}$$

The Guass-Kronrod numerical method is used for calculating this value.

RESULTS AND DISCUSSION

For the different wall temperature cases, where the outer wall is higher than the inner wall ($T_o > T_i$), Brinkman number is positive (*Br*>0), and for the opposite state, i.e. ($T_o < T_i$), Brinkman number is negative (*Br*<0). Figures 2 and 3 show the effect of Brinkman and Deborah number on dimensionless temperature distribution for positive and negative values of Brinkman number. The upper and lower part of diagram are related to *Br*>0 and *Br*<0 respectively; both parts show similar behaviors, but their direction from inner to outer wall is vice versa. As it is apparent from Figure 2, the temperature profile is nearly linear in shape for small values of Brinkman number; however, for *Br*>2, it exhibits a maximum value within the annular gap, because by increasing Brinkman number, the magnitude of viscous dissipation term increases in Equation 5. Also, at high values of Brinkman number, the heat flux would switch from heating to cooling near the hot wall, which means that cooling happens in both walls. It is because of the significant effect of viscous dissipation in the region close to the walls in comparison with the central region of annulus. However, due to imposing constant temperature to the walls, this behavior occurs.

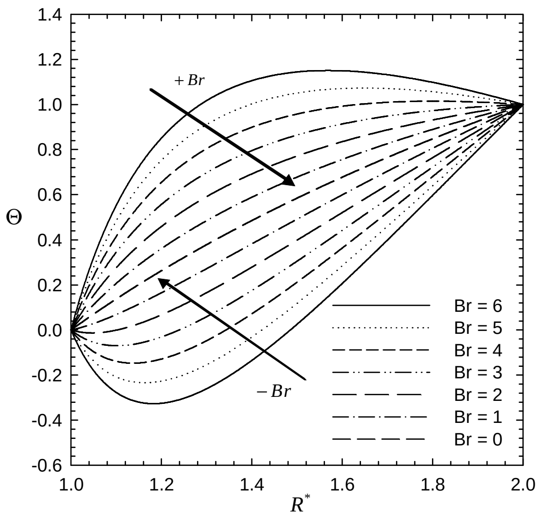

Figure 2: Effect of Brinkman number |*Br*| on the temperature profile; $T_i \neq T_o$, α=0.2, Π=2, β=0, and *De*=1.

As can be seen in Figure 3, the effect of decreasing Deborah number on temperature distribution is similar to the effect of increasing Brinkman number, because by increasing Brinkman number, the viscous dissipation increases; but when Deborah number increases the viscosity decreases due to shear-thinning behavior of this type of fluid and as a result viscous dissipation decreases.

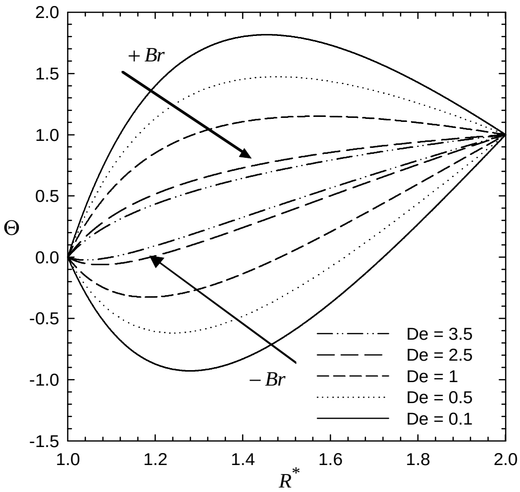

Figure 3: Effect of Brinkman number (*De*) on the temperature profile; $T_i \neq T_o$, α=0.2, Π=2, β=0, and |*Br*|=6.

Temperature profile is presented in Figure 4 for different values of velocity ratio (β) ranges, i.e. 0-2. Increasing β acts as decreasing Brinkman number or increasing elasticity. For β=2, the angular velocity of the outer cylinder (Ω_o) is equal to the inner cylinder (Ω_i), i.e. no relative angular motion exists between the outer and inner cylinders; hence velocity gradient is constant and according to Equation 5, the term of viscous dissipation is eliminated and temperature distribution is similar to Figure 2 and for *Br=0* it becomes nearly linear.

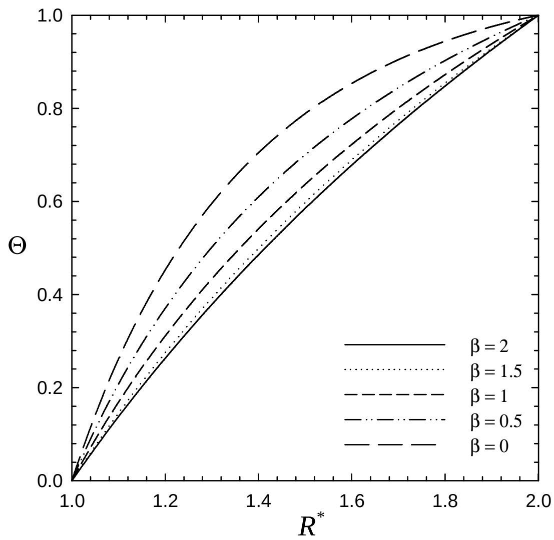

Figure 4: Effect of velocity ratio (β) on the temperature profile; $T_i \neq T_o$, α=0.2, Π=2, *Br*=1, and *De*=0.1.

Figures 5 and 6 show the effect of Brinkman and Deborah number on inner Nusselt number. Figure 5 and Figure 6 are related to positive and negative Brinkman respectively. In Figure 5, by increasing Brinkman number, Nusselt number increases because intensive viscous dissipation causes more internal heat generated, which needs to be removed; therefore, Nusselt number will grow. Furthermore, as can be seen in Figure 2, by increasing elasticity, temperature gradient on the inner wall (wall heat flux) is decreased. Also, because of steeper velocity profiles, the thermal resistance is reduced. Hence the difference between the wall and bulk temperatures is decreased, and as for normalized temperature expression, the dimensionless bulk temperature is decreased too. The decrease in wall heat flux by elasticity growth is more than the decrease in temperature difference, and thus Nusselt number is reduced. For positive Brinkman numbers, the minimum temperature is on the inner wall and, according to definition, normalized bulk temperature, and subsequently Nusselt number, should always be positive. Therefore, Nusselt curve is monotonic forever. But For negative Brinkman numbers a singular point will be seen in Nusselt curve (Figure 6). At low Brinkman numbers, because wall heat flux and normalized bulk temperature are both positive, the Nusselt number will be positive; but by increasing viscous dissipation, Nusselt value first reaches zero and then will be negative. This occurrence is relative to the cooling phenomenon on the hot wall, which causes temperature gradient to become zero and then negative as mentioned above. In addition, by increasing Brinkman bulk temperature rises, but wall temperature is constant; therefore, this temperature difference reduces and approaches zero and, as a result, singularity occurs in the Nusselt curve. For higher values of Brinkman, the bulk temperature will be higher than the wall temperature and thus the sign of Nusselt is changed again.

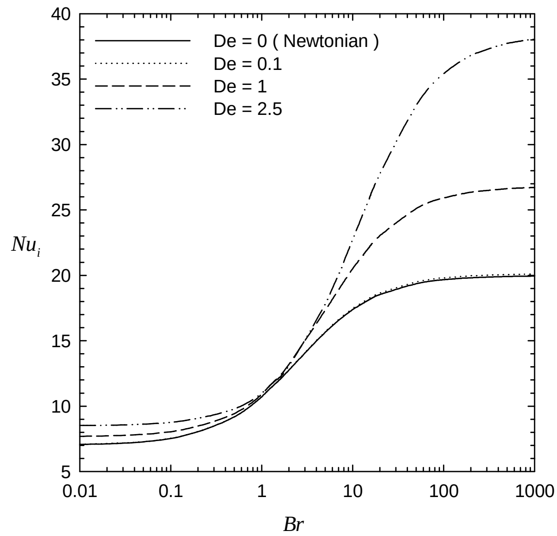

Figure 5: Variation of the inner wall Nusselt number with the Brinkman number (*Br*>0) and *De*; $T_i \neq T_o$, α=0.2, Π=2, and β=0.

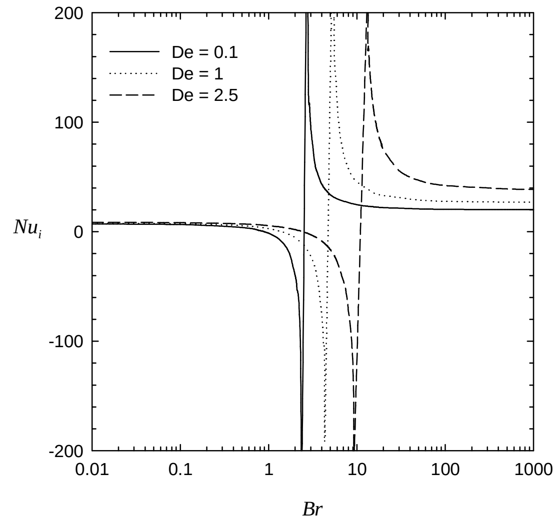

Figure 6: Variation of the inner wall Nusselt number with the Brinkman number (*Br*<0) and *De*; $T_i \neq T_o$, α=0.2, Π=2, and β=0.

Figure 7 shows the effect of Deborah number and mobility factor on Nusselt curve for the positive values of Brinkman number. In this figure, similar to Figure 4, the Nusselt number rises by increasing elasticity level and viscous dissipative effect is remarkable. It is evident that the elasticity increases by increasing α; as a result, the effect of α on heat transfer is similar to the effect of Deborah number.

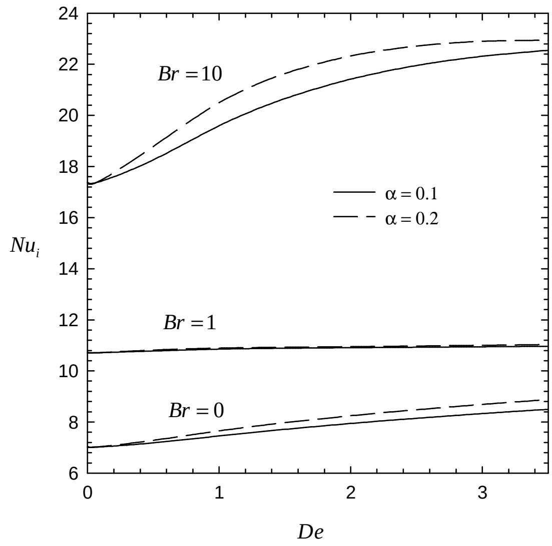

Figure 7: Variation of the inner wall Nusselt number with the Deborah number (*De*); α=0.1, 0.2, $T_i \neq T_o$, Π=2, and β=0.

Figure 8 shows the Nusselt number profiles as function of velocity ratio (β) for different Brinkman numbers. Increasing velocity ratio (β) reduces temperature gradient but increases bulk temperature; hence, according to Equation 20-a, Nusselt number is decreased. For all the Brinkman numbers, the magnitude of minimum Nu_i is the same (≈ 4.3589) and occurs at β=2.

For the identical wall temperatures, as can be seen from Figure 9, the magnitude of Nusselt number on the inner wall is greater than the outer wall, because the temperature gradient of the inner cylinder is higher than the outer ones. By increasing elasticity, the Non-dimensional temperature gradient and normalized bulk temperature are decreased, which leads to a reduction in the numerator and denominator of Nusselt number. However, the effect of the reduction on the denominator is greater than the numerator and increases Nu_i. While for the outer wall Nusselt (Nu_o) the trend is the opposite and effect of decreasing the temperature gradient reduces Nu_o. Additionally, Nusselt number is independent of Brinkman number. By changing Brinkman number, the non-dimensional temperature gradient and normalized bulk temperature vary. However, despite these variations, the Nusselt number remains constant.

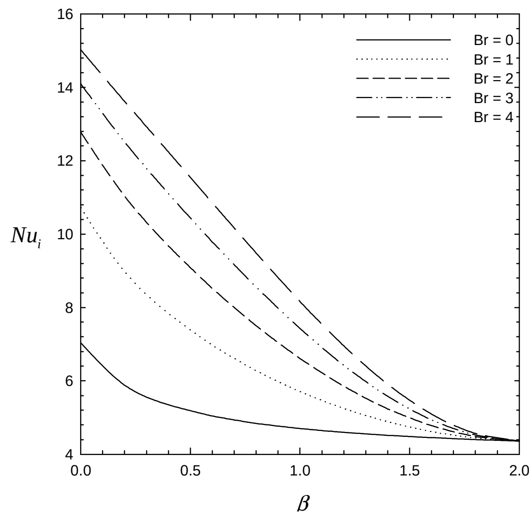

Figure 8: Variation of the inner wall Nusselt number with velocity ratio (β) and *Br*; $T_i \neq T_o$, Π=2, α=0.2, and *De*=0.1.

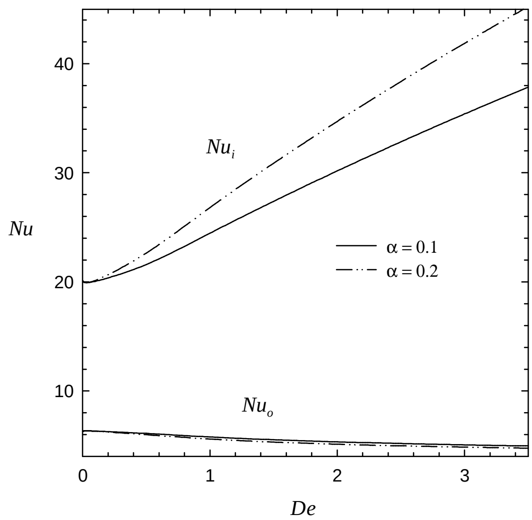

Figure 9: Variation of the inner wall Nusselt number with the Deborah number (*De*); α=0.1, 0.2, $T_i \neq T_o$, Π=2, and β=0.

CONCLUSIONS

In this study, forced convection heat transfer was investigated for steady state, laminar, and purely tangential flow of nonlinear viscoelastic fluids obeying the Giesekus constitutive equation between concentric annulus. Flow inside the annular gap is induced by the relative rotating motion between the inner and the outer cylinders. Thermo-physical properties were assumed independent of temperature and axial heat conduction was negligible. The proposed boundary conditions imposed constant wall temperatures and two states, relative to the different and identical wall temperatures, was considered. The effects of viscous dissipation along with elasticity effect (Deborah dimensionless group and mobility factor) and velocity ratio have been investigated on Nusselt number and non-dimensional temperature profile. The results showed a significant influence of these parameters on heat transfer. The results represented that when inner and outer wall temperatures were identical, the Nusselt number was independent of viscous dissipation.

NOMENCLATURE

Br	Brinkman number
c_1, c_2, c_3	Integration constant in Equations 13 and 15
De	Deborah number ($\frac{\lambda u_c}{\delta}$)
D_H	Hydraulic diameter (2δ)
h	Heat transfer coefficient (W/m^2K)
k	Thermal conductivity (W/m k)
Nu	Nusselt number ($2\delta h/k$)
q	Heat flux (W/m^2)
r	Radial distance
R^*	Dimensionless radial distance
T	Temperature (K)
T_{in}	Inlet temperature
V_c	Characteristic velocity ($R_i\Omega_i$)
V_θ	Tangential velocity (ms^{-1})
X	Constant of Equation 15

Greek Symbols

α	Mobility parameter of Giesekus
β	Velocity ratio ($R_o\Omega_o/R_i\Omega_i$)
δ	Annular gap $(R_o - R_i)$
γ	Shear rate tensor (s^{-1})
θ	Tangential coordinate
Θ	Dimensionless temperature
η	Zero-shear viscosity (Pa.s)
τ	Stress tensor (Pa)
Π	Radius ratio (R_o/R_i)
Ω	Angular velocity ($rad.s^{-1}$)

Superscripts

T	Transpose of tensor
*	Refers to dimensionless quantities

Subscripts

b	Refers to bulk value
i	Refers to inner cylinder
o	Refers to outer cylinder
w	Refers to wall value
I	Refers to $T_i \neq T_o$
II	Refers to $T_W = T_i = T_O$

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